STATISTICAL TESTING
OF
BUSINESS-CYCLE THEORIES

II

BUSINESS CYCLES
IN THE
UNITED STATES OF AMERICA
1919-1932

BY

J. TINBERGEN

LEAGUE OF NATIONS
Economic Intelligence Service
GENEVA
1939

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PROSPERITY AND DEPRESSION

A theoretical analysis of cyclical movements, by Professor Gottfried von Haberler.

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First published in 1937. Thoroughly revised, the new edition has been enlarged by an additional chapter which deals with the recent discussions centring around Mr. Keynes' *General Theory of Employment, Interest and Money.*

As in the previous edition, the first part of the volume contains a systematic analysis of existing theories, while in the second part an attempt is made to construct a comprehensive synthesis of the nature and causes of the business cycle. The book does not, of course, claim to be in any way final, for in this field economic research is constantly advancing, but it prepares the ground for further progress in the attainment of knowledge on points still demanding clarification, and particularly for the statistical analyses attempted in the series of publications entitled *Statistical Testing of Business-cycle Theories.*

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CONTENTS

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PREFACE

This volume on business cycles in the United States of America is the third of a series giving the results of an enquiry into the problem of the recurrence of periods of economic depression upon which the League has been engaged for some years.

The enquiry has been divided into two stages. The first was to examine existing theories with a view to ascertaining what they had in common, the points at which differences of opinion arose and, in so far as possible, the causes of those differences; the second, to confront these theories with the historical facts — to subject them, in so far as these facts can be quantitatively expressed, to statistical analysis, and in so far as they cannot be so expressed, to compare them with the recounted records of the past.

The first stage was completed with the publication of a book, of which Professor Gottfried von Haberler was the author, entitled *Prosperity and Depression*, a revised and enlarged edition of which has just appeared; the second was initiated by the publication this year of an introductory volume[1] by Professor J. Tinbergen, in which the statistical methods which it was intended to employ were explained. In the present volume, also written by Professor Tinbergen, with the assistance of Dr. J. J. Polak, the post-war data for the United States have been employed for the purpose of subjecting to statistical test, as originally proposed, certain of the theories summarised and expounded by Professor von Haberler.

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[1] *Statistical Testing of Business-cycle Theories*, Vol. I: *A Method and its Application to Investment Activity*, League of Nations, Geneva, 1939.

The phenomena of the trade cycle are complex, various forces acting and reacting on each other and constituting in the aggregate a sort of vital organism. In order to understand the manner in which this organism functions, an elaborate system of mathematical analysis is required. The system employed is briefly described in the introduction to this volume and in somewhat greater detail in Professor TINBERGEN's earlier volume.

This is not the place to enter upon any discussion of methodological problems; but it may be well to draw attention here at the outset to one point. The system of analysis employed cannot do more than submit preconceived theories to statistical test. The economist, and not the statistician, must in the first place indicate what, in the light of logical reasoning from ascertained facts, would appear to be the probable causal relationships. The statistician can then examine, with the statistical data and the mathematical tools at his disposal, which of the possible combinations of causes indicated seems in each particular case to give the best fit. He cannot do more than that. It is, indeed, for this very reason that the enquiry has been conducted in two stages — first, an analysis of theories, and secondly, a statistical testing of those theories.

But in practice in the process of testing, in the selection of each one of the "explanatory" factors employed in the various diagrams in this volume, problems of pure economics necessarily arose for consideration before the mathematical analysis could be attempted.

Owing to the nature of this problem and the consequent complexity of the form of analysis employed, a considerable part of this book will present serious difficulties to the non-mathematical reader. His attention may therefore be directed to the introduction, to the conclusions contained in Chapter VII and to the diagrams, which, with the key contained in Appendix A, are largely self-explanatory.

The results obtained can, of course, claim no sort of finality; they relate to one country only, and to a relatively brief period of time during which the economic structure was undergoing very rapid changes. It is proposed to supplement them by a parallel study for the United Kingdom. But it is hoped that it may prove possible, while this parallel study is being conducted, to subject to closer investigation certain points in connection with the work already done, to employ in certain cases monthly or quarterly data instead of annual data and to check certain results regarding post-war events by studies of previous experience.

The manuscript of this volume has been sent to a number of statisticians and economists in different countries who have been good enough to comment on it. In addition, it has been possible to arrange for meetings of small groups of experts to discuss methods and results at various stages of the work. Thanks are due to all those who have been good enough to help by their criticism and advice.

A. Loveday,
Director of the Financial Section
and Economic Intelligence Service.

Geneva, July 1939.

INTRODUCTION

Multiple correlation method.

If one tries to understand the causation of business-cycle phenomena, one is almost invariably led to questions of the type: why did a given economic phenomenon — say, investment activity — fluctuate as it did? Most economists would agree that, generally, a number of "causes" are present, which may all be formulated as changes in some other phenomena. A fall in investment activity may be caused by a fall in profits, or an increase in interest rates, or a change in confidence, and so on. There is less unanimity about the relative strength of these causes in various circumstances. In attempting to find evidence on this relative strength, economic reasoning may be helped and completed by statistical analysis. The ordinary elementary methods of statistical analysis are, however, sufficient for this task only in special cases. One of these ordinary methods consists in looking for months or quarters or years in which only one of the assumed causes has shown a large change, the others remaining about constant. This is, however, a very uncommon case which seldom occurs. Another elementary method is the splitting-up of figures into partial figures — say, general investment activity into investment activity in special branches. The applicability of this method of course depends very much on the statistical material available. But, even apart from that, the splitting-up of the material, however useful, is not sufficient in a considerable number of circumstances. It very often happens that two or more causes are at work even in every subdivision of a phenomenon. In such circumstances, this method is clearly insufficient.

In addition to these elementary methods, a more advanced one — the method of multiple correlation analysis — is available which enables the investigator to find out, in a number of cases, the relative strength of various influences working on the same variable. If, for example, economic reasoning suggests that the fluctuations in variable v (investment activity) depend on fluctuations in the "explanatory" variables Z (profits), q (price of investment goods), m (interest rate) and l (wage rate), then by this method the so-called regression coefficients φ_1, φ_2, etc., can be found by which the variables Z, q, m, etc., have to be multiplied in order to get an expression $v^* = \varphi_1 Z + \varphi_2 q + \varphi_3 m + \varphi_4 l$,[1] the fluctuations of which come as near as possible to those of v. These coefficients can be determined only approximately, their accuracy or significance depending on a number of circumstances which we cannot enumerate now. The details of this method have been given in the preceding publication in this series[2] and need not be repeated here. Numerous results are discussed in the following chapters, and these will serve as examples of the method.

The following features may, however, be shortly recapitulated with a view to a proper understanding of our work.

1. A priori *considerations.*

The method essentially starts with *a priori* considerations about what explanatory variables are to be included. This choice must be based on economic theory or common sense. If *a priori* knowledge regarding the lags to be taken is available, these may be specified also. In many cases, for example, reactions are so quick that only lags of zero length are acceptable. If no such *a priori* knowledge is available, lags may be tried according to the same principle as coefficients — *i.e.*, by finding what lags give the highest correlation. This may be done either by trial and error — when the number of possibilities is quite small — or systematically, by introducing lagged and unlagged explanatory variables (*e.g.*, Z_{-1} and Z) and finding the regression coefficients for these two variables. The relative magnitude of the coefficients will characterise the relative importance of lagged and unlagged influences.

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[1] As all variables will be expressed in terms of deviations from their average value over a certain period (in our case 1919-1932), no constant term is needed.

[2] *Statistical Testing of Business-cycle Theories*, I: *A Method and its Application to Investment Activity*, League of Nations, Geneva, 1939.

2. Lags and annual figures.

It has sometimes been doubted whether short lags can be determined at all from *annual figures.* It is maintained that quarterly or monthly figures would be better or even indispensable. No doubt the latter, when available, contain more information. Since, however, for the most relevant variables the more frequent figures are far less complete than the annual ones (*e.g.*, for profits, investment activity, consumption outlay, capital gains, etc.), and are in addition seasonal, and since some of the accidental movements have already been automatically smoothed out in the annual figures, the latter have been thought preferable. The determination of lags shorter than one year is still possible if the chief fluctuations of the series show periods materially longer than one year. For business cycles this is clearly the case. Further, it must not be overlooked that most lags are by their nature averages of distributed lags,[1] and that the use of annual figures rightly brings in — although admittedly in a rough way — an influence of more remote events. In any case, the significance of the average lags found to exist may be tested along much the same lines as that of single regression coefficients; and in many cases they are found to be significant within the limit of a few months. Only in the case of strongly curvilinear relations will the procedure become inaccurate for years of extreme values.

3. Constant regression coefficients.

Except in a few cases, the equations have been chosen linear, with coefficients that are constant in the course of time. The use of linear relations means much less loss of generality than is sometimes believed. In the case of small variations in variables (v, Z, q, m and l in our example), it can even be proved mathematically that there is no loss of generality at all.[2] In the case of bigger variations, however, it is possible to refine the method

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[1] A notion introduced by Professor Irving Fisher.

[2] It is a well-known mathematical proposition that almost any function f (Z, q, m, l) may be developed in a series which, for small intervals of the variables, can be reduced to a linear expression. And if the coefficient φ_1, with which Z acts on v, itself depends on a new variable x, then it follows that $\varphi_1(x)Z$ may, for small intervals of the variables, also be developed into a linear expression in x and Z.

when necessary. This may be done by introducing as new variables any functions of the explanatory variables, *e.g.*, m^2 or $\frac{l}{m}$ or $\frac{Z}{m}$, according to what the economist would expect to be the relevant combination.[1] On the other hand, it is interesting to note that there are astonishing examples [2] of good fits obtained with constant coefficients and linear equations, which suggest that this type of relation is more frequent than is often believed.

It goes without saying that any regression coefficient found for a market or a group of markets represents only an average for all individuals included, and cannot be applied to problems concerning one individual.

4. Statistical significance.

In order to test the accuracy of results, *statistical tests of significance* must be applied. These have been discussed in the preceding volume in this series, quoted above. The danger threatening the accuracy of our results is especially that of multicollinearity. The simplest form of multicollinearity consists of a high degree of parallelism between two of the explanatory series. In more complicated cases, it may consist of a high correlation between any one of these series and a combination of some others. If such a situation occurs, the separate regression coefficients cannot be determined, though certain combinations of coefficients will still be determinable.[3] The opinion is often expressed that

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[1] Examples of curvilinear dependence will be found in sections 3.5, 4.4 and 4.8 of this study.

[2] *E.g.*, the relation between unemployment and marriages (1870-1913) in *Vierteljahrshefte zur Konjunkturforschung*, Sonderheft 21, Berlin, 1931 (P. LORENZ, " Der Trend "), page 18; the demand curve for beef in the Netherlands (1876-1912), in H. W. METHORST and J. TINBERGEN, " Les recherches relatives à la conjoncture au Bureau Central de Statistique des Pays-Bas ", *Revue de l'Institut international de Statistique*, 1934, I, page 37; the "explanation" of interest rates in the United States before the war in Warren M. PERSONS, " Cyclical Fluctuations of the Ratio of Bank Loans to Deposits ", *Review of Economic Statistics* 1924 (VI), page 260; the "explanation" of world shipping freight rates from 1880 to 1911 in J. TINBERGEN, " Scheepsruimte en vrachten ", *De Nederlandsche Conjunctuur*, March 1934, page 23.

[3] *Cf.* equation (2.1). It may be noted that the knowledge of such combinations is helpful only in so far as periods are analysed in which these intercorrelations are present (*cf.* Vol. I, page 32).

these cases must be frequent in business-cycle research, since all relevant variables show more or less parallel cycles.[1] In the United States, in the period studied here, this was not the case. Some of the reasons for this lack of parallelism are:

(*a*) Interest rates and some other monetary series are much influenced by gold stock fluctuations which are not at all parallel to the general cycle;

(*b*) Commodity prices seem to have come into the region of inelastic supply much more in 1920 than in 1929; they showed very high peaks in 1920, but not in 1929;

(*c*) Share prices showed the reverse behaviour: they were very high in 1929, but not in 1920.

It goes without saying that if some explanatory factor has not changed at all in the period studied, its influence cannot be determined. If it changed only slightly, its regression coefficient may be uncertain. Extrapolation of such results for large variations in the factors concerned is therefore not permitted. For problems of stabilisation, where the aim is to obtain smaller fluctuations, this does not seem to be a serious restriction.

5. Economic significance.

Apart from the purely statistical tests, there are *economic tests* of the significance of the coefficients. The most important one is that of their algebraic sign, which in most cases the economist knows on *a priori* grounds. Sometimes further tests are available concerning the absolute magnitude of one coefficient or the relative magnitudes of several coefficients, occasionally even of different equations. Examples will be found in sections (2.1), (3.3). (3.4), (3.5), (4.3), (4.6).

6. Direct relations.

The word "cause" has been used in the preceding paragraphs to indicate proximate causes only. This means that the economic considerations upon which the relation tested is based must be directed towards finding, as far as possible, "*direct causal relationships*". The variables in the relation must be directly connected either in the

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[1] The author is indebted to Professor R. Frisch of Oslo University for a number of important remarks on this matter, some of which have been used in what follows.

minds of some persons (*e.g.*, through the reaction of the consumer to a given income and price) or by some definition (*e.g.*, value of sales equals volume times price). This is not always possible if the strictest sense of "direct" is kept to. Investment activity may be linked up directly with profit expectations, and these are hardly measurable. The next step connecting profit expectations with actual profits and some other variables may then also be included, and investment activity may be "explained" both by actual profits and by some other variables. The more, however, such combinations of successive steps can be avoided in the formulation of relations, the better. This combination may always be undertaken afterwards — in fact, it forms the very important next step in our work — but the more explicitly it is done, the better. By keeping to this principle, one obtains relations with what Professor Frisch calls[1] the maximum degree of "*autonomy*" — *i.e.*, relations which are as little as possible affected by structural changes in departments of economic life other than the one they belong to. It is clearly the task of economic analysis to indicate the nature of those direct causal relationships.

Complete system of equations.

Returning to the example chosen as our starting-point, it will be clear that, in order to understand the mechanism of business cycles, further steps are necessary. Suppose, for example, that a successful application of multiple correlation analysis shows that the main cause of a given decrease in investment activity was a decrease of 20% in profits, we shall then want to know what caused this decrease in profits. We shall want to find an indirect, a "deeper", cause of the fall in investment activity, which at the same time is a proximate cause of profit fluctuations. This could be done by applying the same method to profits (Z) as to investment activity (v). Still further steps may be necessary: Z may depend partly on the value of total consumption (U), and U must therefore be investigated. If the method can be applied in all cases in which we are interested,

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[1] In private correspondence with the author.

we get an increasing number of relations, representing the network of causal connections forming the business-cycle mechanism, with an increasing number of variables (*i.e.*, time series representing economic phenomena). If we are to understand the mechanism as a whole, we must continue this procedure until the number of relations obtained equals the number of phenomena the course of which we want to explain. We should not be able to calculate, say, n variables if we had only $n-2$ or $n-1$ relations; we need exactly n. Such a system of as many relations as there are variables to be explained may be called a *complete system.* The equations composing it may be called the *elementary equations.* The word "complete" need not be interpreted in the sense that every detail in the complicated economic organism is described. This would be an impossible task which, moreover, no business-cycle theorist has ever considered as necessary. By increasing or decreasing the number of phenomena, a more refined or a rougher picture or "model" of reality may be obtained; in this respect, the economist is at liberty to exercise his judgment. A conclusion about the character of cyclic movements is, however, possible only if the number of relations equals the number of phenomena (variables) included. (The remark may be made here that there is no separate or special variable representing "the cycle" which has to be included in the elementary relations. It is by the mechanism itself that all variables included are compelled to perform cyclic changes.)

An example.

It is perhaps useful at this point to add a few remarks on the nature of a complete system of relations which has to explain business cycles. These remarks can best be made in connection with the concrete example of a very simple system.

Suppose, first, that the value, V_t, of investment goods produced during the period t depends in a linear way on profits one time period (of four months) earlier, Z_{t-1}:

$$V_t = \beta Z_{t-1} \tag{0.1}$$

Both variables are measured as deviations from some "normal", and β is a constant.

Suppose, further, that consumption outlay U_t is the total of:

(i) total wages L_t;

(ii) a term $\varepsilon_1 Z_{t-1}$, indicating that profits Z_t are only partly consumed, the marginal propensity to consume ε_1 being a constant, while there is a lag of four months;

(iii) a term $\varepsilon_2 (Z_{t-1} - Z_{t-2})$, indicating that speculative gains also influence consumption outlay. Speculative gains are supposed to be proportional to the rate of increase $Z_{t-1} - Z_{t-2}$, since share prices are assumed to be a linear function of Z_t and since a lag is again assumed to exist.[1]

We therefore get a second equation:

$$U_t = L_t + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2}) \qquad (0.2).$$

Finally, there is an equation telling how profits Z_t are calculated:

$$Z_t = U_t + V_t - L_t \qquad (0.3).$$

Dynamic features.

Now the system of the three equations (0.1), (0.2) and (0.3) is a "dynamic" system in Frisch's sense, since, in some of the relations ((0.1) and (0.2)), variables appear relating to different time periods. If this were not so — *i.e.*, if all lags were zero and the "speculative" term in (0.2) did not exist — no endogenous cycles could occur. In fact, in such circumstances, the system would be:

$$V_t = \beta Z_t \qquad (0.1').$$
$$U_t = L_t + \varepsilon_1 Z_t \qquad (0.2').$$
$$Z_t = U_t + V_t - L_t, \qquad (0.3').$$

which, after substitution of (0.1′) and (0.2′) in (0.3), gives the equation:

$$Z_t = (L_t + \varepsilon_1 Z_t) + \beta Z_t - L_t$$

or:

$$Z_t (1 - \varepsilon_1 - \beta) = 0 \qquad (0.4').$$

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[1] Since it is only an example we are giving here, details need not be discussed. By comparison with our results in the following chapters, it will be found, however, that in many respects our assumptions are near to reality.

Since ε_1 and β are constants and $\varepsilon_1 + \beta \neq 1$, the only solution is $Z_t = 0$, meaning that the system always shows the same value of profits (Z_t being the deviation of profits from some "normal") and, through (0.1′) and (0.2′), of V_t and $U_t - L_t$ also. No cycles would occur unless the extra-economic "data" determining the "normal" levels showed cycles.

It is quite different, however, in the case of the "dynamic" system (0.1), (0.2), (0.3). The simple structure of the equations still easily permits a substitution of (0.1) and (0.2) in (0.3), leading to a final equation:

$$Z_t = (\beta + \varepsilon_1)\, Z_{t-1} + \varepsilon_2\, (Z_{t-1} - Z_{t-2}),$$

which may be written:

$$Z_t = (\beta + \varepsilon_1 + \varepsilon_2)\, Z_{t-1} - \varepsilon_2 Z_{t-2}.$$

Realistic values for β, ε_1 and ε_2 being 0.2, 0.4 and 1, respectively, we get:

$$Z_t = 1.6\, Z_{t-1} - Z_{t-2} \qquad (0.4).$$

Determinants of the movements.

This equation is of quite a different type from (0.4′). It enables us to calculate Z_t once we are given the values for Z_{t-1} and Z_{t-2}. But then, knowing Z_t and Z_{t-1}, we are again able to calculate Z_{t+1}, and so on. The following table is an example, where Z_0 and Z_1 have been chosen as 0 and + 5 respectively:

$t =$	0	1	2	3	4	5	6	7	8	9	10
$Z_t =$	0	+5	+8	+7.8	+4.5	−0.6	−5.5	−8.2	−7.6	−4	+1.2

The movements we find for Z_t appear to be cyclic. It can easily be ascertained that the actual movement depends on two sorts of given numbers:

(i) the "initial" values of Z_t, in our case Z_0 and Z_1;

(ii) the coefficients of the final equation (0.4), in our case $(\beta + \varepsilon_1 + \varepsilon_2)$ and $-\varepsilon_2$.

The initial values more or less represent what are usually called disturbances from equilibrium; and the coefficients the structure of society. A change in consumption habits would affect ε_1 and ε_2; a change in investment attitude would change β, as would changes in the relative importance of, for example, investment and consumption as a consequence of technical progress. It should be added that the coefficients may also be changed as a consequence of policy, and the problem of finding the best stabilising policy would consist in finding such values for the coefficients as would damp down the movements as much as possible. This will be attained, for example, by small values for the coefficients. *The outstanding importance of the numerical values of the coefficients may be clear from these few considerations.* In fact, it seems difficult to prove by pure reasoning alone — *i.e.*, without knowing anything about the numerical values of the coefficients — whether or not any given theory explains or does not explain cyclic movements. This may be demonstrated by two further numerical examples:

Example A: $\beta = 0.6$, $\varepsilon_1 = 0.8$, $\varepsilon_2 = 1$.
Final equation: $Z_t = 2.4\, Z_{t-1} - Z_{t-2}$.

The type of movement found for any initial value of Z_0 and Z_1 is non-cyclic, with values of Z at an increasing distance from the original values.

Example B: $\beta = 0.2$, $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.1$.
Final equation: $Z_t = 0.9\, Z_{t-1} - 0.1\, Z_{t-2}$.

The type of movement is non-cyclic, with a tendency to return to values $Z_t = 0$ after a short time.

No theory is therefore determinate unless the values of the coefficients in a complete system of equations describing it are known, at least approximately.

Testing of theories.

How, then, can business-cycle theories be tested statistically with the aid of the technique just described? The procedure consists of at least two stages: First, the explanation that a given theory provides for each of the variables of the economic system may be tested by the method of multiple correlation analysis, and secondly, it may be tested whether the system of numerical values found for the "direct causal relations" (or what comes nearest to them) really yields a cyclic movement when used in the final equation.

This may be clarified by indicating the two ways in which an unfavourable result for any theory may be found. First, it is possible that the explanation given for the fluctuations of any of the variables might prove to be poor; and, secondly, it might happen that, although these explanations were not too bad, the combination of the elementary equations would not lead to a cyclical movement.

Apart from these two ways in which a theory may fail, there is the third — already mentioned above — that the theory may prove to be incomplete — *i.e.*, that it contains less relations than variables to be explained — or indeterminate, in that it does not indicate from what other variables each variable depends and in what way.

Strictly speaking, there are very few, if any, "literary" theories that are complete and determinate in the above sense. Most of them — as will be seen from Professor HABERLER's study — emphasise some special relations, often without dealing with most of the others. Practically no single theory can therefore be used for a joint explanation of all the variables included in this statistical study. Nevertheless, many of these "literary" theories may prove highly useful in that they throw light on one detail or a number of details which are indispensable for a right understanding of the business-cycle phenomenon. They must, however, be combined, as Professor HABERLER also points out, and the most efficient way would seem to be to combine all theories open to statistical testing and to test them by means of the system of relations just described.

The present publication is one of the first attempts to construct such a complete system on a statistical basis.[1] The

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[1] In recent times, a number of models of the sort discussed have been constructed (*e.g.*, by AMOROSO, CHAIT, FRISCH, KALECKI, LUNDBERG, ROOS and others); but they have not been based on statistically tested relations, except in part. Models based on statistically determined relations are to be found in J. TINBERGEN, *An Econometric Approach to Business-cycle Problems*, Paris, 1937, and in E. A. RADICE, "A Dynamic Scheme for the British Trade Cycle, 1929-1937", *Econometrica*, January 1939.

character of the work involved is necessarily twofold. It is chiefly of a statistical nature, and to that extent consists in finding the quantitative importance of the chief factors causing fluctuations in each of the variables studied. Carrying out this task presupposes, however, that economic theory — or perhaps several competing economic theories — has indicated what the chief factors are. In this field much still remains to be done. The indispensable minimum of this work which is required in order to make the statistical part of the enquiry possible at all has also been included in this report. This may have led, at some points, to a choice which would not be approved by all economists. Clearly, this cannot be avoided, and the only excuse is that all details of the analysis have been indicated exactly.

Plan of the work.

The first five chapters will contain the description and justification of the relations assumed, and tested statistically, between the phenomena considered as important. In order to treat the matter systematically, these relations are, as far as possible, subdivided into four types, well known in economic theory:

Definitional relations;
Demand equations;
Supply equations;
Income formation equations.

Some relations of another type will be added at suitable places.

Chapters VI and VII are concerned with the resolution of the complete system of equations and the conclusions which can be drawn therefrom, in respect of certain theories and of general characteristics of the business cycle.

The period studied is 1919-1932.

CHAPTER I

DESCRIPTION OF THE MODEL. DEFINITIONAL RELATIONS

(1.0) General Introduction

The construction of a model such as the one to be described here is, in many respects, a matter of trial and error. Exactly what variables are to be included and what neglected is not known beforehand; it only becomes apparent as the work progresses. Starting with some phenomenon of central importance to cyclical movements — as, for example, investment activity — it will first be asked what factors are important in the explanation of this variable; next, what variables are important in the explanation of these explanatory factors; and so on. This procedure must be continued until a number of relations is obtained equal to the number of variables which are considered to require explanation.

It would not serve much purpose to conduct the reader through all the incidental difficulties and errors, some of them at least unavoidable, which beset the course of the reasoning. It seems better to give a rounded-off picture of what has finally been arrived at as the most concise representation of the model. This picture has to start with a list of the phenomena included — a list which in some sense may seem illogical or arbitrary. The best course, therefore, seems to be to present the material in such a way that the reader can easily pick out any variable or relation in which he is specially interested, and can study in whatever order seems to him logical and useful the relations which are here formulated and tested.

The symbols introduced have as far as possible been chosen according to the following rules:

(i) All variables representing money amounts are indicated by capital letters.

(ii) Prices and physical quantities are indicated by small letters.

(iii) Coefficients — in their statistical aspect: regression coefficients; in their economic aspect: elasticity coefficients — are indicated by Greek characters.

(iv) The time period to which a variable relates is indicated by an inferior figure or index t to the right; in so far as no confusion is to be feared, inferior letters are also used for other distinctions, but not figures.

(v) Value symbols and physical symbols relating to the same sort of commodities, etc., are indicated by the same letter (*e.g.*, V and v).

(vi) Related variables are indicated by letters close to each other in the alphabet.

(vii) As far as possible, the same symbols are used as in some previous publications by the author.

A list of variables which may be consulted with any page of the text, will be found in Appendix A. Unbarred symbols represent deviations from the average value of the variable considered over the period 1919-1932; barred symbols represent these averages, and double-barred ones the "natural values" as found in the sources. The symbol $\int$ placed before any symbol indicates that the cumulant of that variable has to be taken. Therefore:

$$\int u_{1927} = u_{1919} + u_{1920} + u_{1921} + u \ldots . + u_{1927};$$
$$\int u_{1921} = u_{1919} + u_{1920} + u_{1921}; \text{ and so on.}$$

The starting-point of the sum is indifferent, provided that it is before the beginning of the period studied ; for suppose that, instead of 1919, 1915 were taken as the starting year, this would only increase *every* value of $\int u$ by the constant amount $u_{1915} + u_{1916} + u_{1917} + u_{1918}$.

As the usefulness of the choice of the variables becomes clear only in connection with the relations chosen, the discussion of the latter may be undertaken immediately.

The relations, a summary of which is given in Appendix B, are, according to the subject of each, treated in Chapters I-V.

As the definitional relations are the least doubtful ones, they may be treated first, although they form a rather incoherent group of not very interesting relations. Not all are definitions in the true sense of the word : some are a description of the composition of some average or total; others represent the rule of computation of some variable. They could be called non-causal relations, in contrast with, for example, demand and supply relations. Some of them, with, in each case, one of the variables which they link together, have only been introduced for reasons of convenience.

The relations have, moreover, not always been given in their exact form. Sometimes they have been replaced by a linear approximation, which, for that reason, does not fit exactly; this approximation entails considerable simplification for the calculations in which the relations are ultimately used. In these cases, the "tests" therefore concern the degree of approximation obtained by these linear expressions, rather than the relations themselves, which are self-evident. The equations are given in alphabetical order of the first variable included. Their obvious nature makes a very short treatment sufficient in most cases.

1.1: $A = 1.50C^i + 0.90B^i + 0.84n - 18.0m_{Lb}$

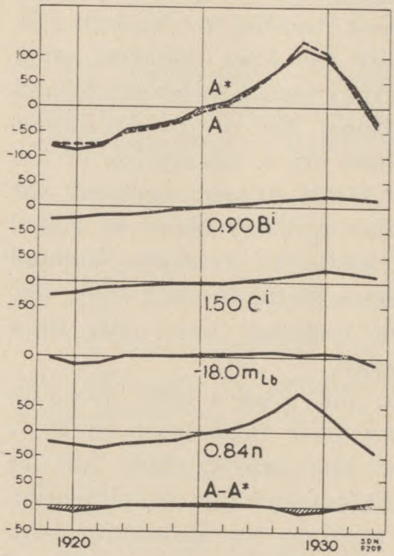
This is an approximation deduced from:

$$\overline{A} = 0.0156\,\overline{C}^i\overline{n} + \frac{c}{\overline{m}_{Lb}}\overline{B}^i,$$

where c is a constant. The total value of assets held by individuals is equal to the value of shares + the value of bonds held by them (*cf.* section (4.7)).

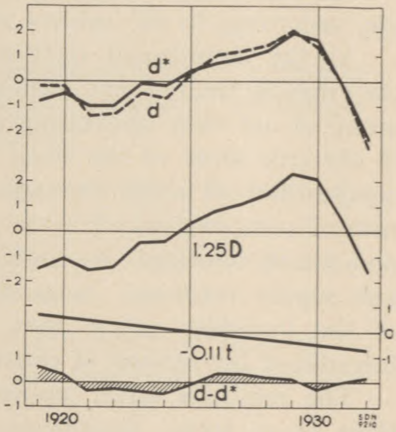
Graph 1.1.

Composition of Fluctuations in TOTAL VALUE OF ASSETS held by Individuals.



Graph 1.2.

" Explanation " of Fluctuations in DIVIDENDS as a PERCENTAGE OF CAPITAL.



1.2: $d = 1.25\mathrm{D} - 0.11\,t$

Here, d represents all cash dividends as a percentage of capital, and D the amount of cash dividends paid to private shareholders.

This relation has been deduced from

$$\bar{\bar{d}} = \frac{100\,\bar{\bar{\mathrm{D}}}'}{\bar{\bar{\mathrm{C}}}}$$

where $\bar{\bar{d}}$ is all cash dividends as a percentage of capital;
$\bar{\bar{\mathrm{D}}}'$ is amount of all cash dividends;
$\bar{\bar{\mathrm{C}}}$ capital, nominal value.

This relation may be written as

$$\dot{\bar{d}} + d = \frac{100\,(\bar{\mathrm{D}}' + \mathrm{D}')}{\bar{\mathrm{C}}\,(1 + \dfrac{\mathrm{C}}{\bar{\mathrm{C}}})}$$

which, by a well-known first approximation, turns into:

$$\bar{d} + d = \frac{100}{\bar{C}}(\bar{D}' + D')\,(1 - \frac{C}{\bar{C}})$$

$$= \frac{100\,\bar{D}'}{\bar{C}} + \frac{100}{\bar{C}}\,D' - \frac{100\,\bar{D}'}{\bar{C}^2}\,C$$

where the second order term has been omitted, which for our figures is certainly admissible. The constant term $\frac{100\,\bar{D}'}{\bar{C}}$ being equal to $\bar{d}$, we are left with

$$d = \frac{100}{\bar{C}}\,D' - \frac{100\,\bar{D}'}{\bar{C}^2}\,C.$$

Here, the second term in the right-hand member is almost a trend, because C is nearly so; in addition, it is very small. Further, D′ will move parallel to D; hence a regression equation between d, D and t has simply been tried, leading to formula (1.2) above.

$1.3\colon E = D + L_C + K_I + K_R + (E_E - E'_F - E''_F)$

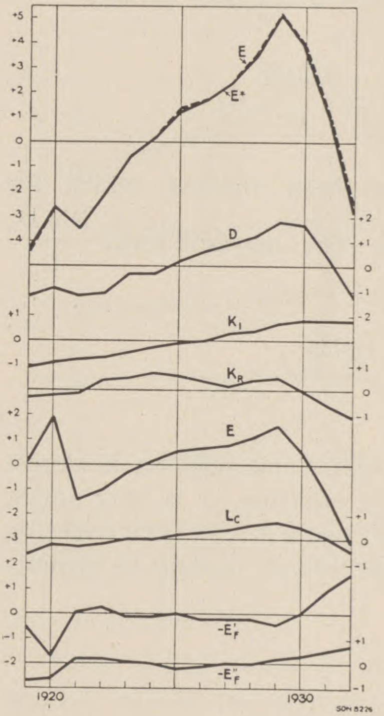
Urban non-workers' income consists of dividends, managers' salaries, interest payments, rent incomes and entrepreneurial withdrawals without farmers' income. As the estimate for the latter may (following the National Bureau of Economic Research)[1] be taken equal to farmers' estimated consumption, it is here represented by $E'_F + E''_F$. The small amount of income from abroad has been neglected. The influence of this neglect is seen in the graph as the difference between the dotted and the full line.

---

[1] *Bulletin 59:* "Income originating in Nine Basic Industries, 1919-1934" by S. Kuznets. New York, 1936. All farmers' savings are considered as business savings. Any net investment farmers are performing is supposed to be paid out of business savings.

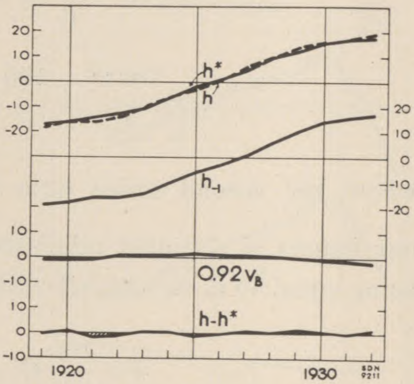
Graph 1.3.

Composition of Fluctuations of URBAN NON-WORKERS' INCOME.



Graph 1.4

"Explanation" of Fluctuations in STOCK OF HOUSES.



1.4: $h = h_{-1} + 0.92 v_B$

The stock of houses at the end of a year is found by adding to the stock at the end of the previous year 0.92 times the volume of residential building during the year. The remaining 0.08 accounts for replacement (estimated according to the figures of WICKENS and FOSTER for the relation between replacement and total building).[1]

---

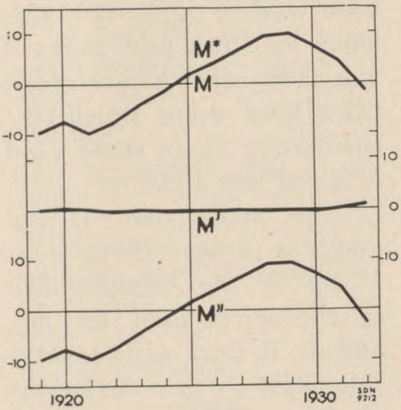
[1] " Non-Farm Residential Construction, 1920-1936 ", *Bulletin 65*, National Bureau of Economic Research, New York, 1937, page 11.

1.5 : $M = M' + M''$.

Total money is equal to the sum of the amount of outside currency (M') + the amount of deposits (M'').

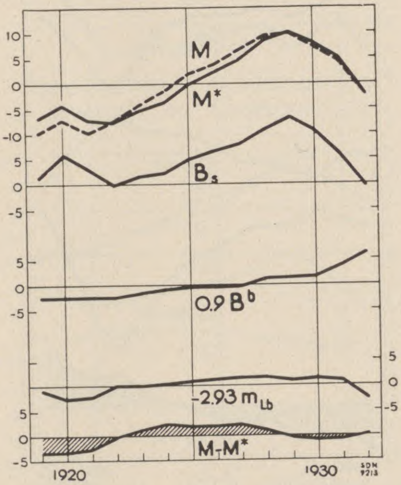
Graph 1.5.

Composition of Fluctuations in TOTAL MONEY.



Graph 1.6.

Fluctuations in ASSETS AND LIABILITIES OF THE BANKS.



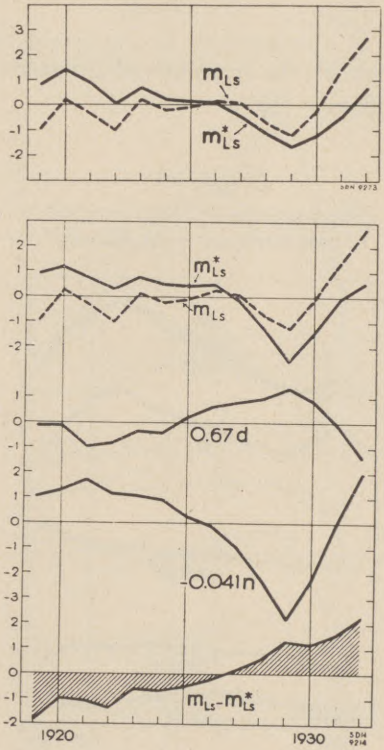
1.6 : $M = B_s + 0.9B^b - 2.93\, m_{Lb}$.

Balance equation for the banks; *cf.* section (4.5).

1.7: $m_{Ls} = 0.67d - 0.041n$

This relation is a simplified form of the definition of $\overline{\overline{m}}_{Ls}$:

$$\overline{\overline{m}}_{Ls} = \frac{100\overline{\overline{d}}}{1.56\overline{\overline{n}}}.$$



Graph 1.7.

" Explanation " of Fluctuations in SHARE YIELD.

Upper part: $\frac{100\bar{\bar{d}}}{1.56\bar{\bar{n}}}$ in deviation from average $= m_{Ls}$

Lower part: $m_{Ls} = 0.67d - 0.041n$.

Here the factor 1.56 has been added, since in 1926, when the stock price index $\bar{\bar{n}}$ was 100, the actual stock price level upon which the calculation of the share yield is based was 156.[1]

The upper part of the graph is to be considered as a test of the compatibility of the series used for m_{Ls} and d; if they were exactly compatible, no deviations should occur. The lower part shows the combined effect of the lack of compatibility and of the linear approximation. Evidently there is, in this case, a danger in using the linear approximation for extreme values like those in 1928, 1929 and 1932.

---

[1] The calculation runs as follows: $\bar{\bar{m}}_{Ls} = \bar{m}_{Ls} + m_{Ls} =$

$$= \frac{100\,(\bar{d}+d)}{1.56\bar{n}(1+\frac{n}{\bar{n}})} = \frac{100}{1.56\bar{n}}\,(\bar{d}+d)\,(1-\frac{n}{\bar{n}}) = \frac{100\,\bar{d}}{1.56\bar{n}} + \frac{100\,d}{1.56\bar{n}} - \frac{100\,\bar{d}n}{1.56\bar{n}^2},$$

neglecting the second order term.

$$\text{Now } \frac{100\bar{d}}{1.56\bar{n}} = \bar{m}^*_{Ls}; \quad \frac{100}{1.56\bar{n}} = 0.67, \text{ and } \frac{100\bar{d}}{1.56\bar{n}^2} = 0.041.$$

* The actual figures show a difference owing to the independence of the sources from which d, n and m_{Ls} haven been taken.

1.8: *Composition of cost of living.*

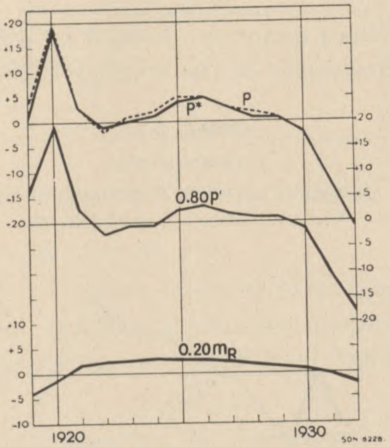
Cost of living is made up of two rather heterogeneous elements — viz., rents m_R and prices of other services and goods p', with weights of 20% and 80% respectively:

$$p = 0.80\,p' + 0.20\,m_R.$$

The "explanation" of m_R and p' is discussed in Chapter III.

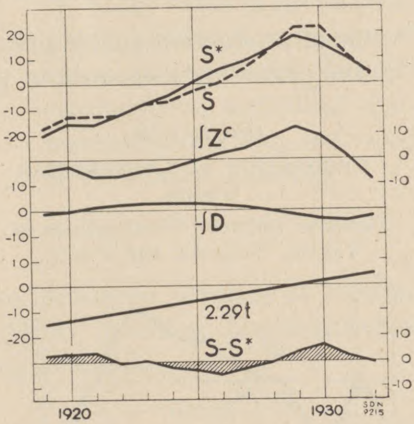
Graph 1.8.

Composition of Fluctuations in Cost of Living.



Graph 1.9.

"Explanation" of Fluctuations in Surplus of Corporations.



1.9: $S = \int (Z^c - D) + 2.29t$

Surplus of corporations depends on cumulated profits ($\int Z^c$) minus cumulated dividends ($\int D$). One would expect simply $S = \int Z^c - \int D$; but, it appears, additional reserves — possibly secret reserves — are constituted, so that the yearly increase in S is larger than $Z^c - D$. If we suppose these additional reserves to be constant,[1] they explain part of the trend. Another

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[1] It was found by correlation calculus that they are not correlated with $Z^c - D$.

part, however, stands as a complement to the cumulation terms. For these being cumulations of the deviations of Z^c and D from their averages, cumulations of the constants $\overline{Z}^c$ and $\overline{D}$ have to be added, which evidently are trends. As $\overline{Z}^c - \overline{D} = 0.5$, a term of $0.5t$ corresponds to $\int(\overline{Z}^c - \overline{D})$. The rest of the trend, $1.8t$, represents the unexplained reserves mentioned above.

1.10: $U = 0.60p + 1.00u$

This relation is the simplified form of the relation

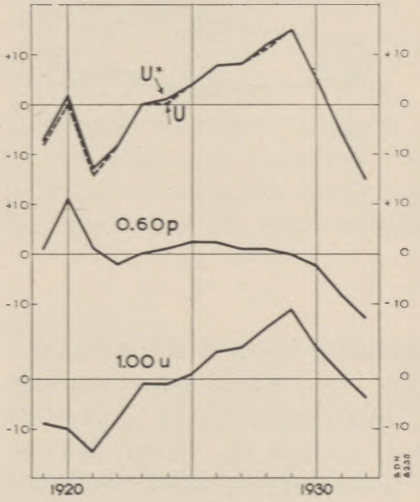
$$\overline{\overline{U}} = 0.01\overline{\overline{pu}},$$

value of production equals price times quantity, divided by 100 as the prices are measured in percentages of the level of 1929.[1]

Graph 1.10.

PRODUCTION OF CONSUMPTION GOODS.

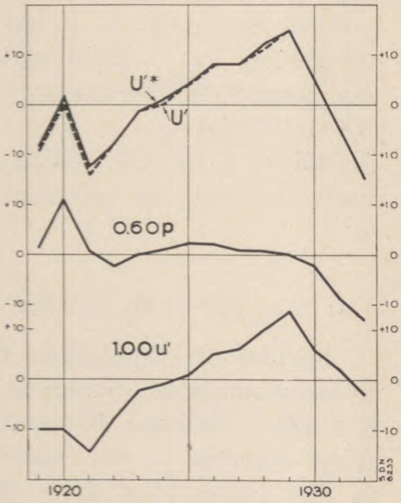
Relation between Fluctuations in VALUE, VOLUME and PRICE.



Graph 1.11.

CONSUMPTION.

Relation between Fluctuations in VALUE, VOLUME and PRICE.



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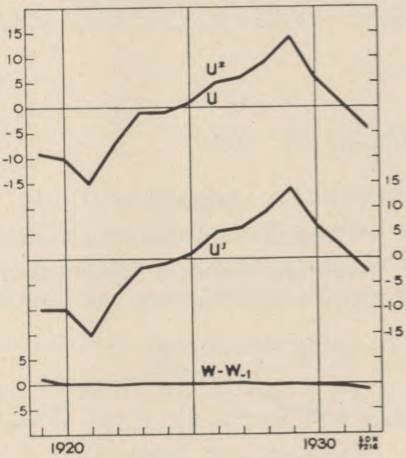
[1] The calculation runs: $\overline{\overline{U}} = \overline{U} + U = 0.01\,(\overline{p} + p)\,(\overline{u} + u) = 0.01\overline{p}\overline{u} + 0.01\overline{p}u + 0.01\overline{u}p$, neglecting the second order term; $0.01\overline{p}\overline{u} = \overline{U}$; $0.01\overline{p} = 1.00$; $0.01\overline{u} = 0.60$.

Graph 1.12.

Relation between Fluctuations in PRODUCTION, CONSUMPTION and changes in STOCKS OF CONSUMPTION GOODS.



1.11: $u = u' + w - w_{-1}$

Production equals consumption + increase in stocks.

1.12: $U' = 0.60p + 1.00u'$

For explanation, see 1.10.

1.13: $V = V' + V_B$

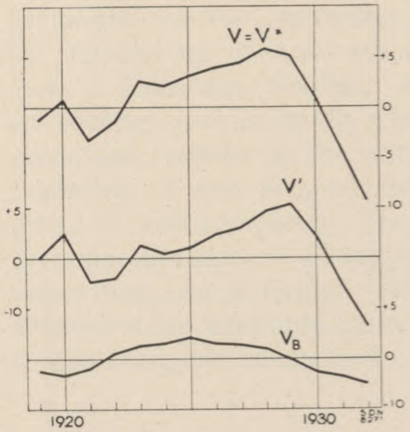
Total value of investment goods produced consists of value of producers' durable commodities, including non-residential building, and value of residential building.

1.14: $v = v' + v_B$

Volume of investment goods produced consists of volume of producers' durable commodities, including non-residential building, and volume of residential building.

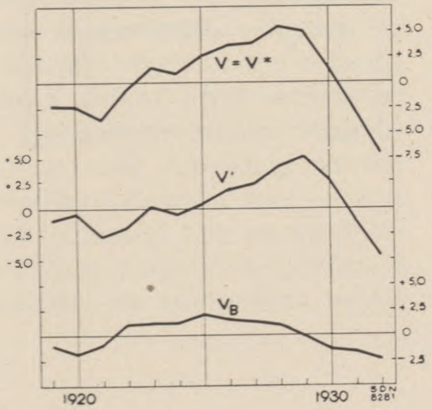
Graph 1.13.

Composition of Fluctuations in VALUE OF PRODUCTION OF INVESTMENT GOODS.



Graph 1.14.

Composition of Fluctuations in VOLUME OF PRODUCTION OF INVESTMENT GOODS.



1.15: $V' = v' + 0.15q$

Same explanation as for 1.10.

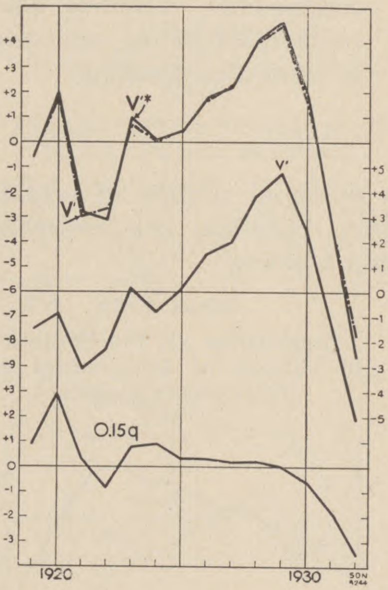
1.16: $V_B = 0.98 v_B + 0.028 q_B$

Same explanation as for 1.10.

Graph 1.15.

PRODUCTION OF PRODUCERS' DURABLE COMMODITIES.

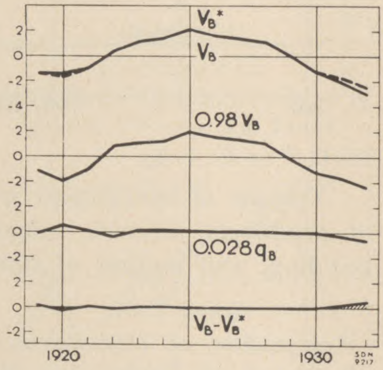
Relation between Fluctuations in VALUE, VOLUME and PRICE.



Graph 1.16.

RESIDENTIAL CONSTRUCTION.

Relation between Fluctuations in VALUE, VOLUME and PRICE.



CHAPTER II

DESCRIPTION OF THE MODEL. DEMAND EQUATIONS FOR GOODS AND SERVICES

(2.0) General Introduction

In this chapter, a number of relations determining the *demand for goods and services* will be discussed. The demand for holding some types of assets will be considered in Chapter IV. The goods and services expressly considered in this study are:

(i) Consumers' goods and services, excluding "housing" services;
(ii) Agricultural raw materials;
(iii) Housing services;
(iv) Houses;
(v) All other investment goods;
(vi) Labour.

The demand for these types of goods and services will not, however, be studied separately. The reasons for this treatment are mentioned below.

Groups (i) and (iii) have been combined as consumers' goods and services, including housing, since the estimates of the demand for each do not seem to be accurate enough to make a distinction possible. A separate study of the demand for housing services on the one hand, and all other consumers' goods and services on the other hand, would require the consideration of two demand functions each depending on the prices of both categories. The combined demand may — as a first approximation — be supposed to depend only on the combined item, cost of living. Moreover, a study of the combined demand is the minimum basis sufficient for any realistic model of business-cycle mechanism.

The demand for agricultural raw materials has not been studied separately, since it may be assumed that it shows a fairly high degree of parallelism with the demand for consumers' goods and services as a whole, given (i) the proportionality between the output of any commodity and the intake of raw materials and (ii) the tendency of consumers to divide their consumption more or less regularly between agricultural and non-agricultural products. On the other hand, there is some cause to disregard any lack of parallelism, for the simple reason that the statistics of stocks of raw materials are not very satisfactory.

Nor has the demand for labour been considered separately. The output of all final goods and services (Groups (i), (iii), (iv) and (v) above) is very exactly parallel with employment as measured by the Federal Reserve Board index of factory employment. Evidently this reflects the fact that production is a linear function of employment for short-run variations in output.

On the other hand, the demand for consumers' goods and services will be split up into four parts — viz.:

(*a*) Demand exerted by non-farmer consumers;
(*b*) Demand by farmers for farm products;
(*c*) Demand by farmers for non-farm products;
(*d*) Demand by dealers corresponding with increases or decreases in stocks.

Although in some respects arbitrary, this subdivision is useful for statistical reasons. In the first place, the factors determining one of these categories of demand will be at least partly different from those determining the others; hence a more exact determination of the coefficients will be possible if they are studied separately. In the second place, the figures for (*c*) and (*d*) have still more the character of estimates than those for (*a*) and (*b*).

In the next chapter a number of supply equations, or their equivalents, will be discussed. This means that, for some categories of goods and services, both the demand and the supply relation will be determined. The well-known question whether, and in what circumstances, a statistical determination of both relations is possible has been touched upon in the preceding volume.[1] In section (3.5), an example is elaborated.

(2.1) "EXPLANATION" OF CONSUMERS' OUTLAY[2]

I. *Theoretical.*

As regards consumers' outlay — in which outlay for the purchase of new houses has not been included — it has been assumed that farmers' outlay for consumption goods is equal to their withdrawals[3] as estimated by Dr. KUZNETS.

The following variables would then, by *a priori* reasoning, seem to be of importance for the explanation of the rest of consumption fluctuations:

Wages and salaries $(L_w + L_s)$;
Urban non-workers' income E;
Capital gains G;
The rate of increase in farm prices $p^f - p^f_{-1}$, or Δp^f, as an indication of speculative profits, which are not included in E but may nevertheless have influenced consumption (agricultural prices have been selected as they are especially subject to speculative influences);
Some measure of the degree of inequality of income distribution, for which PARETO's α has been taken;[4]
Cost of living p;
A trend, standing for slow changes in habits, population growth and changes in population structure.

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[1] Vol. I, pages 62-64.

[2] *Cf.* J. J. POLAK, "Fluctuations in United States Consumption, 1919-1932", *Review of Economic Statistics*, XXI, February 1939.

[3] All their savings being considered as business savings. *Cf.* page 25.

[4] This coefficient measures, in absolute amount, the slope of a curve representing log N_x as a function of log x; where x is income and N_x the number of persons having an income above x.

It has been proved by BORTKIEWICZ that, in general, α is not a very accurate index for distributions deviating from the Paretian; for this reason, the values of α have been tested by comparing them to another index of in equality — viz.: the difference between the median and the average income of the $2\frac{1}{2}\text{‰}$ of the population with the highest incomes. The correlation for this period was very high, and α showed considerable variations (the extremes being 1.39 and 2.04).

The influence of some of these variables, especially E, might be lagged. A lagged influence of G and p is somewhat less probable, as capital gains will be consumed fairly rapidly in so far as they are consumed at all; while the chief influence of cost of living will be that actual prices have to be paid which may differ from the price level upon which the consumption plans were based.

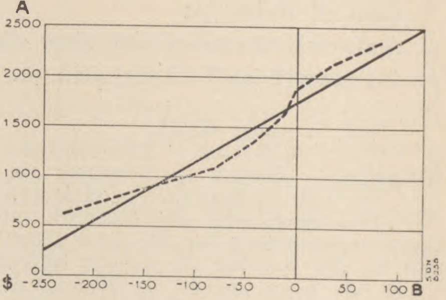
The signs of all coefficients except the trend must be positive. For E and G this will be clear at once; for p, the theoretical possibility exists of a negative influence. A negative influence would, however, mean an elasticity of total consumption which is larger than one, and this will hardly be assumed to prevail by any economist. The significance of Pareto's α being that an increase in α means a decrease in concentration, it seems logical to expect a positive influence of α on consumption.

The two income series $(L_w + L_s)$ and E show a very high intercorrelation. Hence, the coefficients to be obtained for each by including both in a correlation calculation must be expected to be rather unreliable. There are two other ways by which more reliable information might be obtained regarding the two marginal propensities to consume — viz. : (i) to have recourse to knowledge from other sources on the propensity of one of the two income classes, or (ii) to try different reasonable values for one propensity and to see whether the coefficients which result for the other are acceptable.

Graph 2.11.

Amount saved at Various Income-levels.

(Families in New York, Portland and Atlanta, 1936; data from U.S. Bureau of Labor Statistics.)



A = Income.
B = Savings or, where negative, deficit.
- - - - Observed data.
——— Straight line general trend.

Some information about the relation between wages and workers' savings may be taken from family budget statistics, though these statistics give figures relating to families with different incomes at the same moment; and it is not certain that one family, when passing (temporarily) from one income to another, will show the same change in savings. The direction of the deviation between the figures depends on whether savings are a relatively "sticky" item in the budget or not. This in turn will depend on the form of saving. If saving is effected in the form of fixed payments of insurance premia, it may be "sticky"; if small amounts are paid from time to time into savings banks, savings may be more sensitive. From a number of family budget data, represented in Graph 2.11, it would appear that the fluctuations in savings are between 0.15 and 0.20 times the fluctuations in wages.

II. *Statistical.*

In view of these results, a number of correlation calculations have been made where, in each case, the alternative of a fixed coefficient for ($L_w + L_s$) of 1.00 and 0.80 was calculated; the

Case	Variable explained	Regression coefficients								Correlation coefficient
		L_w+L_s*	E	G	Δp^f	α	p	E_{-1}	t	
1 *a*	$U'-E'_F$	1.00	0.78	0.35					0.31	0.992
b		0.80	1.20	0.36					0.18	0.989
2 *a*		1.00	0.75	0.27	0.046				0.32	0.995
b		0.80	1.17	0.26	0.054				0.18	0.994
c		0.95	0.86	0.27	0.048				0.28	0.995
3 *a*		1.00	0.52	0.26		—6.30			0.26	0.995
b		0.80	0.93	0.26		—6.40			0.12	0.993
4 *a*		1.00	0.77	0.35			0.001		0.31	0.992
b		0.80	1.03	0.36			0.069		0.35	0.989
5 *a*		1.00	1.37	0.22				—0.75	0.50	0.993
b		0.80	2.01	0.17				—1.03	0.44	0.991
6 *a*		1.00	0.71	0.28	0.046		0.016		0.36	0.995
b		0.80	0.95	0.27	0.056		0.087		0.41	0.994
c		0.95	0.77	0.28	0.049		0.034		0.37	0.994

* Fixed coefficient.

two values resulting for the coefficients of the other explanatory variables suffice to calculate such values for any other coefficient for $(L_w + L_s)$ by means of a straight-line interpolation or extrapolation. The results are shown in the table on page 37.

The regression coefficient for E, which represents the "partial marginal propensity to consume (in respect to E)" is unacceptable in cases 1*b*, 2*b*, 4*b*, 5*a* and 5*b*, where it is above unity. Cases 3*b* and 6*b* are also hardly acceptable, as they represent a propensity to consume for workers which would be lower than that for the higher incomes. By interpolation, we find that the minimum coefficient for (L_w+L_s), which is higher than the corresponding coefficient for E, is as follows:

In case 1	0.93
,, ,, 2	0.92
,, ,, 3	0.84
,, ,, 4	0.90
,, ,, 5	> 1.00
,, ,, 6	0.87

According to the principles set out above, cases 3 and 5 are both unacceptable for the supplementary reason that they yield a negative coefficient for α and E_{-1} respectively. The remaining cases point to a coefficient for $(L_w + L_s) > 0.87$. The value 0.95 has finally been chosen for the coefficient for $L_w + L_s$.

For G and Δp^f, coefficients are obtained which are only slightly dependent on the choice of the $(L_w + L_s)$ coefficient (the spread between cases *a* and *b* is negligible). The inclusion of Δp^f increases the correlation coefficient to a not unimportant extent (case 2 as compared with case 1). The increase in the correlation by the inclusion of p is immaterial, but its omission is theoretically unsatisfactory. These considerations lead to the choice of an equation which includes as "explaining" variables: $L_w + L_s$, E, G, Δp^f, p and t, with a fixed coefficient for $L_w + L_s$. It has, with the standard errors [1] of the coefficients, the following form:

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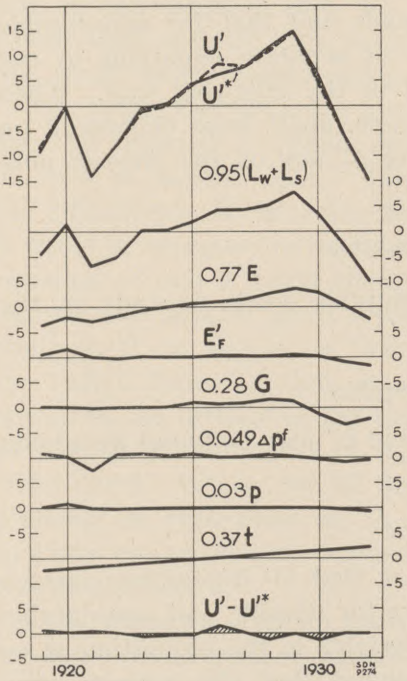
[1] *Cf.* Vol. I. For the calculation of standard errors it has been assumed throughout this publication that the random errors in all observations (to which errors the residuals are supposed to be due) are mutually independent.

$$U' - E'_F = 0.95\,(L_w + L_s) + (0.77 \pm 0.32)\,E + (0.28 \pm 0.13)\,G + (0.05 \pm 0.02)\Delta p^f + (0.03 \pm 0.09)p + 0.37t, \qquad (2.1)$$

where the left-hand member represents urban consumption outlay.

It will be seen that even after the coefficient for $(L_w + L_s)$ has been fixed, that for E is still relatively uncertain; this is principally due to the high intercorrelation between E and p. We are bound to conclude, then, that the values of three coefficients in this equation, those for $(L_w + L_s)$, E and p, cannot be found with a high degree of precision.[1] The consequences for the system as a whole of this interchangeability of the influences of these three variables will be considered in Chapter VI.

Graph 2.1.
" Explanation " of Fluctuations in CONSUMPTION OUTLAY.



The result chosen would mean that workers and lower employees have a marginal propensity to consume of 95%, urban non-workers a "partial marginal propensity to consume" of 77% in relation to " pure income " E, and a "partial marginal propensity to consume" of about 28% of realised capital gains.

This latter coefficient is, however, also rather uncertain, not on the ground of multicollinearity, but because the amplitude of the fluctuations in capital gains has been estimated very roughly.[2]

It should be borne in mind that constancy in the partial marginal propensities does not imply any constancy of the proportion of

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[1] *Cf.* also page 42, note 2.
[2] *Cf.* also page 127.

incomes consumed, *i.e.*, of the ratio $\bar{\bar{U}}' \div (\bar{\bar{L}}_w + \bar{\bar{L}}_s + \bar{\bar{E}} + \bar{\bar{E}}'_F)$, which we may call e. First, when the marginal propensity to consume is smaller than the ratio of the *averages* of consumption and income, e will be lower in a boom than in a depression. Secondly, since capital gains will be high in the *rising* and low in the *declining* phase of the cycle, e has a tendency to behave accordingly. Thirdly, the trend term in the equation means that there is a slow secular increase in e (0.6 % per annum).

It is, of course, possible that the coefficients themselves are not constant either; but, given the nature of the statistical material, it seems almost impossible to obtain reliable information in this respect by the inclusion of more variables; the formula chosen may therefore be considered as about the best possible approximation.

III. *Durable and Non-durable Consumption Goods.*

The demand for durable goods and that for non-durable goods have not been included as separate equations in our system. This may be justified in the following way. When the demand U'_D for durable goods, apart from depending on income Y, depends on their price p_D and on the price of non-durable goods p_N:

$$U'_D = \omega_{11} Y + \omega_{12} p_D + \omega_{13} p_N,$$

and the demand U'_N for non-durable goods depends on the same factors:

$$U'_N = \omega_{21} Y + \omega_{22} p_D + \omega_{23} p_N,$$

then the equation for total demand U' may be found by adding up these two equations:

$$U' = \omega_1 Y + \omega_2 p_D + \omega_3 p_N.$$

This may be understood to mean that U' depends on income Y and some average price index for durable and non-durable goods — viz., an average with weights in the proportion of ω_2 to ω_3. It is not certain beforehand that the average price level p for consumers' goods will show such weights. Since, however, there will be a tendency to some parallelism between p_N and p_D — owing to the general competition on both the demand side and the supply side — there is no serious loss of generality if we replace the theoretically best average having weights ω_2 and ω_3 by our index p.

In the consumption equation (2.1), only "general variables" — *i.e.*, variables bearing on all goods, not on one category alone — occur. This implies the hypothesis that there are no factors bearing especially on durable or on non-durable goods.

Now there is one special feature in the demand for durable goods which may behave contrary to this hypothesis. Demand for durable goods consists of two parts — viz., replacement demand and so-called first purchases. The latter will, in general, depend on much the same general factors as the demand for non-durable goods — income, prices, tastes. The former will, however, depend on earlier purchases of the same goods[1] and will, in the simplest case, be equal to the quantity bought before some definite time period, representing the lifetime of the goods under consideration. (In more complicated cases — viz., where this lifetime is not a definite period, but purchases may be deferred — other determining elements may come in, such as income again. This does not, in theory, increase the difficulties.) If this echo effect proved to be of importance, it would be necessary to take it into account in the consumption equation — and it might then perhaps be useful to treat non-durable and durable goods separately. Now it appears, from a study by P. DE WOLFF on "The Demand for Passenger Cars in the United States", [2] that, at any rate for one commodity, the spread in the lifetime of the individual objects is large enough to smooth out the curve of replacement purchases to a mere trend curve. Hence, for all durable goods together, this will probably be even more so. A study of the year-to-year fluctuations in consumers' demand may for this reason neglect the echo effect.

Yet, though a separate study of the demand for durable and for non-durable goods is not essential to the present system of

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[1] The so-called "echo effect".

[2] *Econometrica* VI (1938), page 113.

equations, this division is of such outstanding general interest that it may be worth while to digress slightly and give an "explanation" of both categories of goods. Apart from the explanatory variables used for U′, it will be necessary, as mentioned above, to include in both equations p_{D} and p_{N}, the prices of durable and non-durable goods respectively. In order not to have too large a number of variables, and in view of the high intercorrelations between some of them, the income series are here combined into two groups — ordinary incomes $(\mathrm{L}_w + \mathrm{L}_s)$, E and $\mathrm{E'_F}$ and speculative incomes G and Δp^f — and in each group the series are weighted according to the coefficients they have obtained in the "explanation" of U′.[1] The results run as follows, with standard errors of the coefficients added:

Series "explained"	Coefficients and standard errors of					R
	$0.95(\mathrm{L}_w+\mathrm{L}_s) + 0.86\mathrm{E}+\mathrm{E'_F}$	$\mathrm{G} + 0.16\Delta p^f$	p_{D}	p_{N}	t	
$\mathrm{U'_D}$	0.16 ± 0.05	0.03 ± 0.03	-0.028 ± 0.038	2.056 ± 0.020	-0.05	0.984
$\mathrm{U'_N}$	0.73 ± 0.09	0.23 ± 0.05	-0.006 ± 0.075	0.069 ± 0.041	0.40	0.996
U′ (by addition)	0.89	0.26	−0.022	0.125	0.35	
U′ (case 2 c)	1.00	0.27			0.28	

The coefficient for the ordinary incomes is, for both groups together, below 1. This is due to the inclusion of p_{D} and p_{N}, which are rather highly correlated with incomes. It will be seen that in case 6*c*, where p is included in the explanation of U′, the coefficient for E is also much lower.[2]

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[1] Here the case with the same explanatory variables as in (2.1) except p, and a coefficient of 0.95 for $\mathrm{L}_w+\mathrm{L}_s$ (case 2*c*) was used.

[2] The four price coefficients make it possible to check the p-coefficient in equation (2.1), if we use the approximation that p_{D} and p_{N} move parallel. The coefficient of p in the "explanation" of U′ is then equal to the average of the sum of the two coefficients for p_{D} and the sum of the two coefficients for p_{N}, weighted according to the relative weights of p_{D} and p_{N} in p multiplied by their relative amplitudes. This yields 0.11, whereas we had found 0.03 ± 0.09 in the case chosen ; both coefficients are, indeed, rather near to zero.

The figures point to the following elasticity of demand with respect to ordinary incomes,[1] price and the price of the competitive category of goods:

Goods	Income elasticity	Price elasticity	
		Own price	Other price
Durable	1.23±0.38	— 1.39±0.53	0.74±0.27
Non-durable	0.81±0.10	— 0.87±0.08	0.01±0.15

Somewhat higher income and price elasticities are brought out for the durable than for the non-durable group, but the significance of both differences is doubtful.

It may be interesting to apply to these data the "Slutsky condition" of the rational, consistent behaviour of consumers, the formulation of which in our symbols would be:[2]

$$\frac{\delta u'_{\mathrm{D}}}{\delta p_{\mathrm{N}}} - \bar{u}'_{\mathrm{N}} \frac{\delta u'_{\mathrm{D}}}{\delta \mathrm{Y}} = \frac{\delta u'_{\mathrm{N}}}{\delta p_{\mathrm{D}}} - \bar{u}'_{\mathrm{D}} \frac{\delta u'_{\mathrm{N}}}{\delta \mathrm{Y}}$$

where Y stands for $\mathrm{L}_w + \mathrm{L}_s + \mathrm{E} + \mathrm{E}'_{\mathrm{F}}$.

Using the figures of the first table, this condition would be:[3]

$$5.4 - (51.8 \times 0.148) = 0.6 - (7.46 \times 0.675).$$
$$-2.0 = -4.5.$$

It will be seen that the coefficients, taken at their face value, do not exactly fulfil the condition. But when we take account of their standard errors,[1] the result becomes:

$$2.0 \pm 4.1 = -4.5 \pm 7.6.$$

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[1] There would not be much sense in calculating average income elasticities with regard to speculative incomes, since their average is, by their very nature, zero or almost zero.

To arrive at one coefficient for the three ordinary income groups, the coefficients obtained for each of them have been weighted according to their standard deviations (relative amplitudes); the weighted marginal propensity to consume for all consumption goods would be 0.924.

[2] *Cf.* H. Schultz, *Theory and Measurement of Demand*, Chicago, 1938, page 621.

[3] The price coefficients are multiplied by 100 since the averages of p_{D} and p_{N} used in the transformation from U'_{D} to u'_{D} and from U'_{N} to u'_{N} are about 1, and not about 100. All coefficients, moreover, are corrected for the small deviations of $\bar{p}_{\mathrm{D}}$ and $\bar{p}_{\mathrm{N}}$ from 100.

It is quite possible, then, that the "true" coefficients do satisfy the condition.

(2.2, 2.3) "EXPLANATION" OF FARMERS' CONSUMPTION

For this part of the investigation, rather rough assumptions have been made, as (i) the part of total income going to farmers is only about 10% and (ii) refinements would require the introduction of some new variables which would complicate the system without improving it very much.

The relative smallness of its fluctuations makes a very accurate consideration of this item unnecessary, while the rather rough estimates available do not seem to lend themselves to any detailed experiments with correlation calculus.

The prevailing factor governing gross as well as net farm incomes and the estimates of farmers' consumption is, of course, farm prices. The volume of farm production, which depends largely, in any case for the period up to 1932, on crop-yield variations, shows only irregular and not very wide fluctuations.

Farmers' consumption consists of two parts: viz., consumption of home-produced goods and of bought goods, the money values of which are indicated by E''_F and E'_F respectively. Both are supposed to depend only on farm prices p^f.

For E''_F this will be clear. For E'_F it means that the elasticity with respect to prices of non-farm products is just 1, which seems probable in view of the relatively low standard of living of the farm population. The formula found is:

$$E'_F = 0.025\,p^f \qquad (2.2).$$

As to E''_F, the formula found, viz.:

$$E''_F = 0.015\,p^f, \qquad (2.3)$$

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[1] Combination of standard errors according to formula:

$$\sigma^2_{(1-2)} = \sigma_1{}^2 + \sigma_2{}^2 - 2\sigma_1\sigma_2 r_{12},$$

where $r_{12} = \dfrac{M_{11}}{(M_{22} \times M_{33})}\,^{\frac{1}{2}}$ (*cf.* Vol. I, pages 142-143).

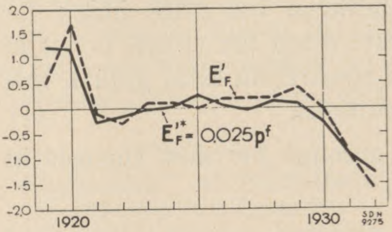
implies that the quantities of farm products retained — viz.,

$$\frac{\overline{\overline{E}}{}''_F}{\frac{1}{100}\,\overline{p}^f} = \frac{1.7 + 0.015\,p^f}{0.97 + 0.01\,p^f} = \frac{(1.7 + 0.015\,p^f)\,(1 - 0.01\,p^f)\text{ [1]}}{0.97}$$

$= 1.8 - 0.002\,p^f$, depend negatively on farm prices, with an average elasticity of demand of — 0.11.

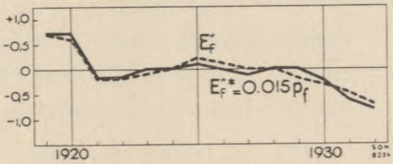
Graph 2.2.

"Explanation" of Fluctuations in FARMERS' CONSUMPTION EXPENDITURE.



Graph 2.3.

"Explanation" of Fluctuations in FARMERS' CONSUMPTION OF HOME-PRODUCED GOODS.



(2.4, 2.5) "EXPLANATION" OF INVESTMENT ACTIVITY

Investment may take various forms, each of which is subject to its own "laws". For the purpose of this investigation, a distinction has been made between:

v' investment in durable producers' goods, including non-residential building;

v_B investment in residential building;

v_w investment in stocks of non-durable commodities (working capital).

Purchases of durable consumers' goods have simply been included in consumption.

The relations which "explain" the purchases of each type of these goods may be indicated as "demand equations for investment goods". As the first publication in this series[2]

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[1] Owing to a well-known mathematical approximation.
[2] Vol. I, Chapters III and IV.

deals especially with these relations, they need only be mentioned briefly here.

The demand v' for *durable producers' goods and non-residential building* has been considered in combination. It has been assumed to depend on:

(i) Profits made in all industries, for which corporation profits Z^c have been taken;

(ii) Share yield m_{Ls}, as an indication of the "interest rate paid" on capital obtained by share issues;

(iii) The price of investment goods q;

(iv) The margin $p - \frac{1}{2}l$ between the price index for finished goods and the wage rate (with the weight it has in costs), as it is often held that, apart from total profits, this margin influences profit expectations.

(v) A trend, in order to account for slow changes in capital intensity of production.

For all variables a lag of half a year has been assumed.[1]

The introduction of share yield as one of the determining factors needs further elucidation. One way of looking at the matter is that, although no yield is contracted when shares are issued, the yield which satisfies investors will depend on the general situation in the share market as represented by the share yield on existing shares. It would not matter, in this train of thought, if the actual yield on new shares were systematically lower than the average yield on old shares, provided it could be maintained that there was a systematic relation in the fluctuations of both.

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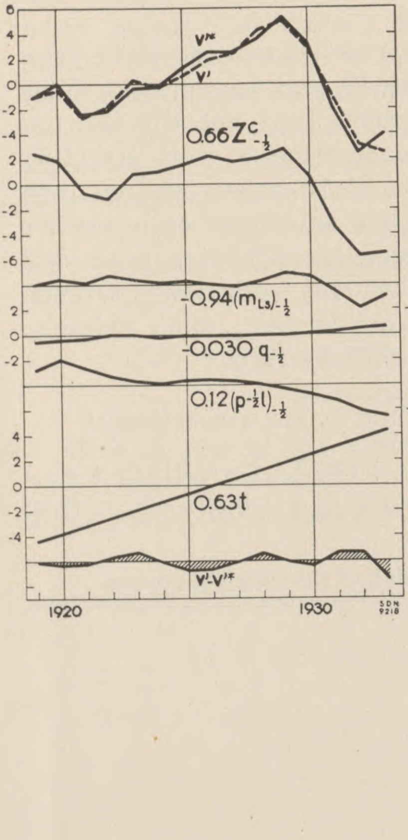
[1] Expressed somewhat more exactly, the lag is a distributed one with an average of half-a-year. In fact, by using annual data, one is only able to apply lags of 0, 1, 2 etc. years, but any combination may be taken which means a distributed lag. The average of these lags, weighted according to the regression coefficients obtained for the term corresponding to each, may be indicated shortly as "the" lag. If, *e.g.*, the following regression equation is found: $v = 0.3\,Z + 0.5\,Z_{-1}$, this weighted average of the lags 0 and 1 is $\frac{(0 \times 0.3) + (1 \times 0.5)}{0.3 + 0.5} = 0.63$.

Another way of interpreting the matter is that the " easiness " with which money is obtained by share issues could be given a numerical expression by the figure of share yields.

Still another way would be to point to the factors "behind " share yield, which fluctuates inversely to share prices and proportionately to dividends. Share prices themselves (*cf.* equation (4.82)) are influenced by both dividends and the rate of increase in share prices. Instead, therefore, of assuming investment activity to be negatively affected by share yield, one could formulate our hypothesis thus: that investment activity is favourably affected by the rate of increase in share prices, favourably affected by share prices themselves (the higher these prices, the higher the issue prices entrepreneurs are able to get), and unfavourably by dividends (which in a sense is the "payment " they are expected to make).

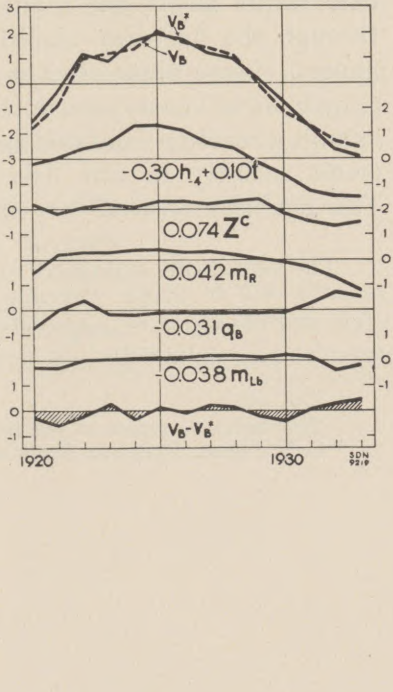
Graph 2.4.

"Explanation" of Fluctuations in DEMAND FOR DURABLE PRODUCERS' GOODS, including NON-RESIDENTIAL BUILDING.



Graph 2.5.

"Explanation" of Fluctuations in RESIDENTIAL BUILDING.



Three other variables were also tentatively included, but, as their regression coefficients were found to be exceedingly small, they have been left aside. These variables are:

(*a*) The rate of increase in consumers' goods' production, in order to account for a possible direct influence of the "acceleration principle";

(*b*) The rate of increase in prices of investment goods, in order to account for a possible speculative attitude.

(*c*) The interest rate for short credits (m_S).

The rejection of these variables has been considered at some length in the preceding volume in this series.[1]

The demand for new dwellings v_B has been assumed to depend on:

(i) Rent level m_R;
(ii) Cost of construction q_B;
(iii) Long-term interest rate m_{Lb};
(iv) Profits Z^c;
(v) Number of houses h;

with a lag of zero for the series (i) to (iv) and one of $3\frac{1}{2}$ years for (v). The first four series may be said to represent direct incentives which work without much lag,[2] but the last one only works slowly and indirectly. It seems to work especially through the financial condition of house-owners who let their houses. Some time after a relative scarceness or a relative abundance of houses occurs, the financial condition of owners will exhibit a reaction; and this again will only work slowly, through credit security in this branch of enterprise, upon building. This has been treated very accurately by Roos.[3]

The equations obtained for v' and v_B are, respectively:

$$v' = 0.33\,(Z^c + Z^c{}_{-1}) - 0.47\,[m_{Ls} + (m_{Ls})_{-1}] - 0.015\,(q + q_{-1}) \\ + 0.06\,[p + p_{-1} - \tfrac{1}{2}\,l - \tfrac{1}{2}\,l_{-1}] + 0.63\,t \qquad (2.4)$$

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[1] *Loc. cit.*

[2] The series v_B refers to the beginning of the building process.

[3] C. F. Roos: *Dynamic Economics*, Bloomington, 1934, pages 69-110.

$$v_{\mathrm{B}} = -0.30h_{-4} + 0.074\mathrm{Z}^{c} + 0.042m_{\mathrm{R}} - 0.031q_{\mathrm{B}} - 0.038m_{\mathrm{L}b} + 0.10t \qquad (2.5).$$

For further details concerning these relations the reader may be referred to the first volume in this series.[1]

The way in which investment in working capital has been treated is somewhat indirect, but, in view of the rather deficient statistics, it is perhaps the best that can be adopted. It consists in regarding all enterprises as though they were integrated, without attempting to deal separately with the various vertical stages of production. This "body" of enterprises shows an output of goods and services in the final stage and an input of factors of production. If production in all stages were exactly synchronised, these factors would only be used for the production of the final goods leaving the "body". Investment in working capital means, however, that, at various places in the "body", stocks of raw materials and intermediate products accumulate — *i. e.*, that, in some earlier stages of the process, more is produced than corresponds to final output. This will reflect itself in a greater application of factors of production, and therefore in a larger total of wages — the other factors being mainly "overhead" factors. Investment in working capital therefore finds its expression in total wages L_w and farm incomes. Because, however, of the rather short series now available for all stocks, it has not been possible to consider separately what factors seem to be important in an explanation of working capital as a whole.

Only investment in stocks of finished consumers' goods may be treated more completely.

(2.6) "Explanation" of Commodity Stocks (Consumers' Goods)

This is one of the least satisfactory parts of the present study, chiefly because of lack of adequate data. It has only been possible to consider the most important causes of changes

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[1] *Loc. cit.*

in stocks. After inspection of the curves, these seemed to be purely technical; they may be formulated as follows:

(i) There is a tendency to hold stocks which are proportional to sales; and

(ii) This tendency is counteracted by unforeseen changes in sales, of which production cannot immediately take account.

The first tendency points to considering as the first determining factor of stocks w the amount of sales u'; the second to including as a second factor the change in sales as compared with those of the previous year; this latter with a negative sign as an increase in sales will, *ceteris paribus*, lead to low stocks. This leads to the formula:

$$\begin{aligned} w &= \Omega u' - \Omega' (u' - u'_{-1}) \\ &= \Omega_1 u' + \Omega_2 u'_{-1}; \; (\Omega_1 = \Omega - \Omega'; \; \Omega_2 = \Omega') \qquad (2.61). \end{aligned}$$

Further, the interest rate and price changes would seem to influence the holding of stocks of finished consumers' goods.

For the series of department-store stocks, a slight influence of the former factor[1] was found; but price changes did not seem to have a marked influence either on this series or on that of stocks of manufactured goods. A final judgment on this question will be possible, however, only when more abundant material is available. After a number of years, the statistics of corporations will certainly yield a very useful contribution; the series of data now available is, however, too short.

The relation (2.61) was tested for department-store stocks, for which it was found to fit very well. The same type of formula was therefore used for the "explanation" of w, for which the relation

$$w = 0.105u' + 0.047u'_{-1} - 0.187(m_S)_{+\frac{1}{2}} - 0.307t \qquad (2.6)$$

was found. The trend was introduced to represent secular

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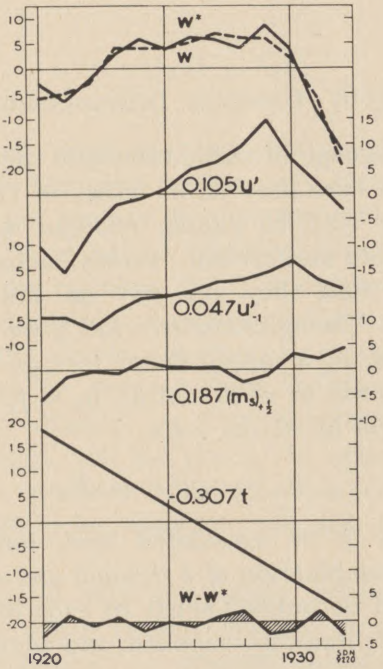
[1] Represented in equation (2.6) by $(m_S)_{+\frac{1}{2}}$, since w represents stocks at the end of the year and m_S is an average over the year.

changes in the habits of holding stocks. It is equivalent to a decrease in stocks of some 4% per annum, which does not seem unreasonable.

Graph 2.6.

"Explanation" of Fluctuations in
STOCKS OF CONSUMERS' GOODS.



CHAPTER III

DESCRIPTION OF THE MODEL. SUPPLY OR PRICE EQUATIONS FOR GOODS AND SERVICES

(3.0) General Introduction

Using the language of static economic theory, the relations to be considered here may be of either of two types. In the first place, they may be *supply relations* which connect up price, quantity sold and certain "supply factors" of which the characteristic is that they act only on the supply side, all "demand factors" being excluded. Let p be the price, u_S the quantity supplied, F_S a supply factor — *e.g.*, unit cost — then a supply relation will be of the form: $u_S = f_1(p, F_S)$. A linear approximation will be of the form:

$$u_S = \omega_{1p}p + \omega_{1S}F_S \qquad (3.01).$$

The relations to be considered here, however, may also result from the combination *of a demand and a supply relation,* which is obtained by putting equal to each other the quantity demanded and the quantity supplied, and then eliminating this quantity.

Calling F_D any demand factor — *e.g.*, income — the demand relation will be of the form: $u_D = f_2(p, F_D)$; with a linear approximation:

$$u_D = \omega_{2p}p + \omega_{2D}F_D \qquad (3.02).$$

To apply both relations to the price actually prevailing and the quantity u actually sold, we have to put the quantity demanded u_D equal to the quantity supplied u_S, and we get, in the case of the linear approximations:

$$\omega_{1p}p + \omega_{1S}F_S = \omega_{2p}p + \omega_{2D}F_D$$

which may be written

$$(\omega_{1p} - \omega_{2p})\, p + \omega_{1S}F_S - \omega_{2D}F_D = 0$$

$$\text{or } p = \frac{\omega_{2D}F_D - \omega_{1S}F_S}{\omega_{1p} - \omega_{2p}} \text{ (supposing } \omega_{1p} \neq \omega_{2p}).$$

In general:

$$p = f_3\,(F_D, F_S) \qquad (3.03).$$

Such a relation, which may for shortness be called a *price equation*, connects the price with supply factors and demand factors, but does not contain the quantity sold. It will be clear that a system of two relations consisting of a demand equation (3.02) and a supply equation (3.01) is equivalent to a system of two relations consisting of a price equation (3.03) and either the demand or the supply equation, since the third equation in each case may be deduced from the two others. It will therefore simply depend on the circumstances which of these three systems will be given. In general the demand equation will be given as such (*cf.* Chapter II), but either the supply or the price equation will also be given (in this chapter).

This procedure is only completely valid for some special types of market which exhibit freedom of supply and — as a necessary counterpart — absolute adaptability of prices. In many modern markets, this is no longer the case. Prices are "sticky" and supply is not entirely free. The demand relation in general remains in existence, although it, too, may, for psychological reasons, not react to prices immediately, but only with a lag. The supply relation takes rather the form of a "price fixation relation" — *i.e.*, of a relation telling on what factors producers or sellers base themselves when fixing the price. This relation contains the same variables as the old supply relation, but price is effect rather than cause and may therefore be lagged behind quantities and supply factors.

Using the symbols just introduced, it could be written in the form

$$p = f'_1 (F_S, u_S) \text{ or}$$

$$p = \pi_F F_S + \pi_u u_S \tag{3.04}$$

which replaces (3.01).

The fixing of price may also be effected by negotiations with the demand side (*e.g.*, the labour market) and thus depend also on demand factors; but in this case demand factors can always be eliminated again by using the demand equation, and therefore the price fixation equation in its first form may still be used.

Some applications of these notions are to be found in the following sections.

(3.1) "EXPLANATION" OF WAGE RATE

The equation introduced here is a price fixation equation which, as has already been said, may be considered as a supply equation for labour, although serious objections can be raised against this terminology, in as much as it presupposes a free market. It has been assumed that wages, if looked at from the workers' standpoint, will depend on —

(i) employment,

(ii) cost of living,

(iii) labour productivity,

(iv) institutional factors, such as the changing strength of trade unions, legislation, etc.

Employment is, as far as its fluctuations are concerned, intimately correlated with volume of production. Therefore $u + v$ has been taken for the first series.

Cost of living p has also been included, whereas, for the period 1919-1932, the two remaining factors are considered as trend factors. (For the extrapolation through 1934, this hypothesis would no longer be valid.)

In view of the stickiness of wages, a lag has been introduced — though this procedure increases the difficulty of applying the ordinary concept of "elasticity of supply". The length of the lag is established by correlation analysis in introducing the wage rate of the following year l_{+1} as one of the "independent" variables.[1] This leads to a regression equation

$$l = 0.52(u + v) + 0.67p - 0.72l_{+1} + 0.89t,$$

which may be written in the form

$$l + 0.72l_{+1} = 0.52(u + v) + 0.67p + 0.89t,$$

or, combining the two terms in the left-hand member of the equation and dividing by 1.72:[2]

$$l_{+0.42} = 0.30\,(u+v) + 0.39p + 0.51t. \qquad (3.1).$$

The lag in wages would thus be about five months. The average "elasticity of the supply of labour" (using this term with the reservations just mentioned) would be $\frac{1}{0.30} \times \frac{\bar{l}}{\bar{u} + \bar{v}}$ = about 4.0.[3]

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[1] It may be added that almost the same result is obtained if one starts with a calculation "explaining" l_{+1} by l, $u + v$, p and t (*i.e.*, when another elementary regression is used).

[2] The formula used is

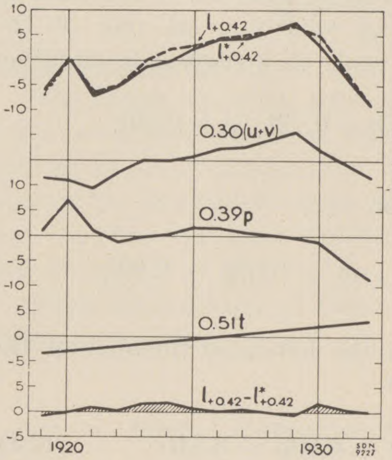
$$\alpha_1 l + \alpha_2 l_{+1} = (\alpha_1 + \alpha_2)\, l_{+\frac{\alpha_2}{\alpha_1+\alpha_2}}$$

which is strictly valid only for a rectilinear development of l during any two consecutive years. For this rather small interval, this approximation is justified.

[3] Here it has been assumed that volume of production $u + v$ and employment vary proportionately; if account is taken of the discrepancy, one must deduct about 20% at most from the above figures.

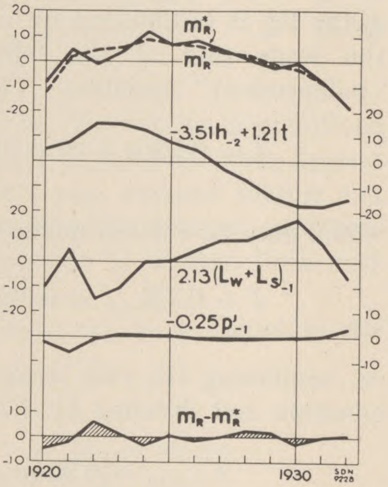
Graph 3.1.

"Explanation" of Fluctuations in WAGE RATE.



Graph 3.2.

"Explanation" of Fluctuations in RENT LEVEL.



(3.2) "EXPLANATION" OF RENT LEVEL

Here a "price equation" has been chosen for the explanation. Rents have been assumed to depend on:

(i) The stock of houses *h* as a supply factor;

(ii) Labour income, L_w+L_s, being the income of the large mass of tenants, as one demand factor;

(iii) Cost of living without rent (p'), the price of the other goods and services competing for the income, as a second demand factor;

(iv) A trend, in a fixed combination with *h*, as an indication of the normal need for houses (*i.e.*, *h* is included in the calculation in deviations from its trend over the period 1910-1935, which is $2.44t$).

A lag of one year and a half for the stock of houses[1] and of one year for the other variables has been found to

---

[1] As in our system these are counted at the end of each year, we have to take h_{-2}.

give the best explanation. The following relation has been obtained:

$$m_{\mathrm{R}} = -3.51\,h_{-2} + 2.13\,(\mathrm{L}_w + \mathrm{L}_s)_{-1} - 0.25\,p'_{-1} + 1.21\,t \quad (3.2).$$

(3.3) "EXPLANATION" OF PRICES OF CONSUMERS' GOODS AND SERVICES, EXCLUDING RENT (p')

Here a price fixation equation has been chosen. The variables included are:

(i) Farm prices, as a cost element of special behaviour to be explained afterwards;

(ii) Wages, as a direct cost element;[1]

(iii) A trend representing changes in labour productivity, which are largely secular.

There might have been reason to include a fourth variable — viz., quantity produced u. This was in fact tried in several ways (with and without a lag), but the results were not significantly different from those without u. As an extremely small influence of u was found in similar researches made for Holland[2] and for the United Kingdom (pre-war period), u was left out entirely, and the fit was still good. This would mean that, in the period considered, the elasticity of supply of manufactured consumers' goods and of consumers' services was infinite. This does not seem unrealistic in view of (a) the overcapacity which

---

[1] Assumed to reflect also mineral raw-material cost. In fact, there is a very close correlation between non-farm raw-material prices and wage rates with a trend (for changing productivity). The chief reason why these other raw materials are not treated separately is that their prices show almost no autonomous fluctuations, as is the case for agricultural products. The general laws of price formation adhered to in this study are also applicable to them.

In addition, their importance to the total cost of living is only very small; food, clothing and services, which account for about 75% of non-rent expenditure, being practically independent of non-agricultural raw materials.

[2] *Cf.* J. TINBERGEN: *An Econometric Approach to Business Cycle Problems*, Paris, 1938.

seemed to exist[1] and (*b*) the tendency to fixed prices (trade marks, etc.).

The relation found with the variables left is:

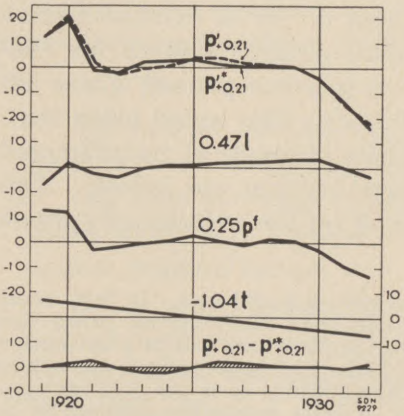
$$p'_{+0.21} = 0.47l + 0.25p^f - 1.04t \tag{3.3}$$

The lag was determined in the same way as for wage rates (*cf.* section (3.1)).

The coefficients obtained for l and p^f are very satisfactory; they correspond fairly exactly to the proportion of direct labour cost and of agricultural raw-material cost in consumers' goods prices. The coefficient for t would seem rather low, corresponding to an annual increase in efficiency of about 1 %, but it is quite possible that other elements work in the opposite direction (such as increasing capital costs, which, in the long run, are reflected in the price).

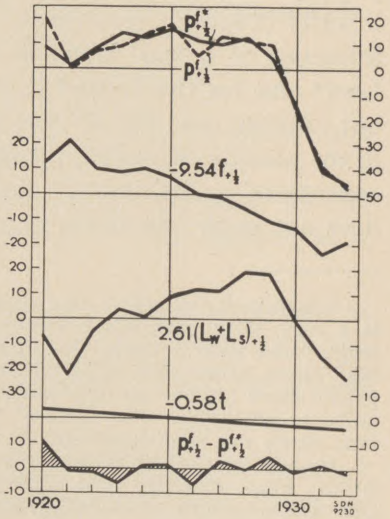
Graph 3.3.

"Explanation" of Fluctuations in COST OF LIVING, excluding Rent.



Graph 3.4.

"Explanation" of Fluctuations in FARM PRICES.



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[1] *Cf. America's Capacity to Produce*, Brookings Institution, Washington, 1935.

(3.4) "EXPLANATION" OF FARM PRICES

For the explanation of farm prices, a price equation (*cf.* general introduction to this chapter) has been used. The following elements were included:

(i) f, the volume of agricultural supply available for the United States market[1], *i.e.*, crops + carry-over — exports.[2] No account has been taken if the world supply of agricultural products, since the greater part of United States farm products are not subjected to competition in the world market; as regards cotton, an important exception, the share of the United States in the world supply is very large.

(ii) Total wages bill $(L_w + L_s)$ as a demand factor.

(iii) A trend, representing the rationalisation in farm production (and other possible trend influences).

No factor was included for direct costs, which are very low in agriculture.

The following regression equation was obtained for the period 1920-1932:[3]

$$p^f = -4.77f + 2.66\,(L_w + L_s) - 2.23t \qquad (3.41).$$

As however, f is highly correlated with t, the coefficients for both factors are very uncertain, as is shown by their standard errors:

$$p^f = -(4.77 \pm 3.67)\,f + (2.66 \pm 0.27)\,(L_w + L_s) - (2.23 \pm 1.37)\,t.$$

---

[1] To be distinguished from the actual market supply, from which it differs by the amount added to stocks.

[2] Imports should not be added here, because they are of a special character and not competitive with United States production (coffee, rubber, etc.).

[3] For all series, crop year figures are used; they are therefore indicated in the graph by the suffix $+\frac{1}{2}$. It seemed expedient not to start the calculation before the middle of 1920, as the guaranteed minimum price for wheat instituted in August 1917 was not repealed until July 1st, 1920 (*Yearbook of Agriculture*, 1921, page 141).

Hence it seems useful to consider these coefficients in the light of knowledge from other sources.[1]

As indicated under (3.0), a price equation may be considered as the result obtained by eliminating from a demand equation and a supply equation the quantity exchanged. Denoting this quantity, which is equal to the market supply, by x^f, we can write these two equations:

$$\left.\begin{array}{ll} \textit{Demand:} & x^f = -\varphi_1 p^f + \varphi_2 (\mathrm{L}_w + \mathrm{L}_s) \\ \textit{Supply:} & x^f = +\varphi_3 p^f + \varphi_4 f + \varphi_5 t, \end{array}\right\} (\text{all } \varphi\text{'s} > 0), \quad \begin{array}{r} (3.42) \\ (3.43) \end{array}$$

from which we eliminate x^f:

$$p^f = \frac{-\varphi_4}{(\varphi_1 + \varphi_3)} f + \frac{\varphi_2}{(\varphi_1 + \varphi_3)} (\mathrm{L}_w + \mathrm{L}_s) - \frac{\varphi_5}{(\varphi_1 + \varphi_3)} t, \qquad (3.44)$$

which is the general form of (3.41).

If we express all series in percentage deviations from their average, we may describe the φ's as follows:

φ_1 : price elasticity of demand;
φ_2 : income elasticity of demand;
φ_3 : elasticity of supply;
φ_4 : proportion of a positive or negative excess of available supply reflected in the actual market supply;
φ_5 : percentage cost decrease *p.a.*, divided by 100.

In order to reduce the limits of the coefficients for f and t, additional information on $(\varphi_1 + \varphi_3)$ and φ_4 *or* φ_5 is sufficient; of the latter two, φ_4 may be chosen as the coefficient on which most knowledge is available.

The fluctuations in market supply differ from those in available supply by changes in stocks. Graph 3.41 compares the latter two series for three cereals and cotton, the major United States farm products for which changes in stocks are important. It is seen that there is a rather close correlation between both

---

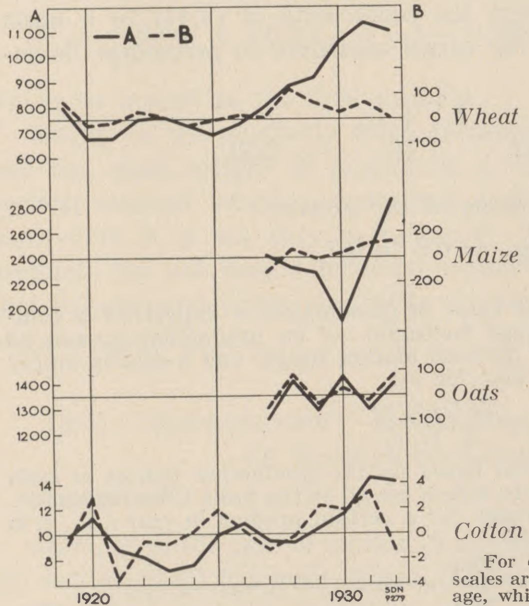
[1] Since $\mathrm{L}_w + \mathrm{L}_s$, which is the only endogenous variable in the explanation of pf, has a fairly certain coefficient, this supplementary analysis is not necessary from the point of view of the systematic cyclical forces; it is only necessary to estimate correctly the influences of changes in crops on the other variables of the system.

series for wheat (up to 1928),[1] oats and cotton, but hardly any for maize (corn). From these data it was deduced that the fluctuations in market supply constitute, on an average, the following percentages of the fluctuations in available supply:

Wheat [2]	40%
Maize	100%
Oats	33%
Cotton	0%
All other farm products . . .	100%

Graph 3.41.

Changes in Stocks of Farm Products compared with Available Supply.



A. Available supply: crop + carry-over — exports. Left-hand scale.

B. Increase in stocks. Right-hand scale.

For each commodity, the A and B scales are the same, except for the average, which is taken at the same level for both curves. Cereals are expressed in millions of bushels; cotton, in millions of bales.

In order to obtain an average value for φ_4, these values for the individual commodities are weighted according to the relative amplitude of the available supply of each commodity

---

[1] From then on, the very large and — until 1933 — increasing stocks presumably could not react to the increased available supply in the same way as before 1929, when stocks fluctuated around a constant level.

[2] 1919-1928; *cf.* note 1.

(in values, at the price of the base year 1927).[1] This yields a value of 0.8 for φ_4.

Similarly, an average elasticity of demand has been calculated [2] on the basis of the elasticities found for twelve important agricultural commodities by Professor SCHULTZ,[3] supplemented by estimates for fruits and nuts, vegetables, poultry and eggs, and dairy products. An average of about 0.5 was found for φ_1.

The elasticity of supply (φ_3) of agricultural products must be very low in relation to the short-term reactions of suppliers that are considered here; it may be taken at 0 or, say, 0.20.

To be able to make use of these more direct estimates for φ_1, φ_3 and φ_4, we convert the coefficients of (3.41) to a basis which corresponds to the series measured in *percentage* deviations from average:

f	$L_w + L_s$	t
-0.62	1.18	-2.52

If we take, on the basis of the above:

$$\varphi_1 + \varphi_3 = 0.5 \text{ to } 0.7,$$

---

[1] Calling the base-year value of (production — export) of a commodity j, U_j; the percentage fluctuation of its production σ_j; and its coefficient for the relation between market supply and available supply φ_j, we find the average value for φ_4:

$$\varphi_4 = \frac{\Sigma_j \, U_j \, \sigma_j \, \varphi_j}{\Sigma_j \, U_j \, \sigma_j}.$$

The σ_j measure has been based on the production indices of each product or group of products, which are all on the basis 1924-1929 = 100. Calling the value of this index for a certain product in year i, u_i, σ_j is calculated with the formula:

$$\sigma_j = \sum_{1920}^{1932}{}_i \, | (u_i - u_{i-1}) | - (u_{1932} - u_{1919})$$

where the second term represents a rough correction for the trend in u.

[2] The formula for averaging the individual elasticities η_j must take account of the degree of fluctuation of the market supply (at the base year value). It runs:

$$\varphi_1 = \frac{\Sigma_j \, U_j \, \sigma_j \, \varphi_j \, \eta_j}{\Sigma_j \, U_j \, \sigma_j \, \varphi_j}.$$

No account has been taken, in this formula, of cross elasticities (*cf.* section (2.1)). But most of Professor SCHULTZ's elasticities are also found without taking account of other prices than that of the particular commodity studied.

[3] H. SCHULTZ, *op. cit.*

we find (*cf.* (3.44)):

φ_4	φ_2 [1]	φ_5
0.31 to 0.43	0.59 to 0.83	1.26 to 1.76

It follows that the coefficient for f in (3.41) is much too low. As, however, its standard error is large, we may attribute to f in (3.41) a considerably different coefficient without really impairing the degree of correlation.[2] A value at twice the coefficient [3] found in (3.41) would be about the minimum compatible with the value of 0.8 for φ_4. The introduction into the correlation calculus of this fixed coefficient for f leaves that for $(L_w + L_s)$ practically unchanged:

$$p^f = -9.54f + 2.61(L_w + L_s) - 0.58t, \qquad (3.4)$$

which we accept as the final formula.

Owing to the relatively small influence of f on farm prices and the small weight of exports in f, we may disregard the cyclical element which is contained in these exports, and the more since it is not very pronounced. The demand for farm products has not been considered separately, it being assumed that this demand varies parallel to the variations in demand for all consumers' goods.

(3.5) "EXPLANATION" OF FLUCTUATIONS IN PRICES OF INVESTMENT GOODS

I. *Theoretical*

Here, as in the case of section (3.3), a price fixation equation (*cf.* section (3.0)) has been chosen for the explanation.

The chief variables included are:

---

[1] On the basis of data from M. LEVEN, *c.s.*, *America's Capacity to Consume* (the Brookings Institution, Washington D.C., 1934, pages 87-88), an elasticity of the demand for food of about 0.5 could be calculated for the lower income classes (which consumed, in 1929, 84% of all food). This figure roughly tallies with the value found for φ_2, and so confirms the estimates for $(\varphi_1 + \varphi_3)$.

[2] In the case chosen, R is 0.975 as compared with 0.979 in case (3.41).

[3] Or 1.3σ above that coefficient.

(i) The variable cost per unit of product, represented by wage rate l;

(ii) The volume of production v';

(iii) A trend, representing the course of overhead cost, technical development, etc.

Since we are considering all enterprises, which are here regarded as being vertically amalgamated, practically the only element in variable cost will be variable labour cost. Its influence on price may be estimated on *a priori* grounds, which in general seem safer than any other basis. Labour cost may be estimated at about 50% of prices of investment goods. This does not correspond, however, to direct labour only, but to all labour. From the figures of the Federal Reserve Board, it may, moreover, be estimated that a 10% increase in production of durable goods is accompanied by a 7% increase in hours[1] of work. Marginal labour cost seems, therefore, to be about seven-tenths of average labour cost for average production. The price increase corresponding to a 10% increase in wages will therefore, in the short run, be equal to $5 \times 0.7\% = 3.5\%$.

The volume of production may represent, in the language of the more "practical" investigator, the strength of the seller in the market.[2] The higher the sales, the larger the addition to direct cost which the seller is able to charge. It is a well-known fact that, in times of severe depression, many enterprises in these branches are making prices only a little above variable cost, while it is only in better times that they are able to earn their overhead cost and profits. The coefficient with which volume of production enters into the equation is closely related to the elasticity of supply, which will be calculated later.

No special attention has been given to the price movements of individual metals. It is not impossible that these movements

---

[1] Hours being estimated by multiplying the employment index by the quotient, weekly wages over hourly wages (National Industrial Conference Board figures). *Cf.* J. Tinbergen, "Profit Margin, Investments and Production" (Dutch), *De Nederlandsche Conjunctuur*, November 1935.

[2] Especially since in these markets most production is to order and over-production therefore practically impossible.

are in part also due to changes in world stocks, but since these stocks themselves depend on production with a certain lag,[1] which is already included in the explanation, it seemed advisable not to take stocks as a separate variable. Moreover, the individual fluctuations in the prices of particular metals practically disappear in a weighted average of these prices (*cf.* graph 3.51;[2] the weight chosen is the value of the world production of each metal in 1930). The average shows practically the same movements as q.

For the "explanation" of q, the period 1919-1932 cannot be considered as a whole, owing to a marked difference in market organisation between the years 1919 to about 1923 on the one hand, and the period after that year on the other. Not until about 1923 could the iron and steel industry, which had been very strongly organised many years before the war, again effectively control the price fluctuations of its products.[3] In the first five years after the war, the price formation of q may have shown other characteristics than in the more monopolised period. The following differences might be expected:

(1) The re-monopolisation may have effected a higher general level of q;

(2) It has very probably diminished the amplitude of the cyclical fluctuations;

(3) It may, in particular, have prevented very rapid price rises when production has been very near to capacity; *i.e.*, it may have prevented bottle-necks.

On the other hand, it is reasonable to assume that the price formation of q has not changed, from the one period to the other, with regard to:

---

[1] *Cf.* L. M. Lachmann and F. Snapper: "Commodity Stocks in the Trade Cycle", *Economica* V, pages 435-454, November 1938.

[2] Page 66.

[3] *Cf.* A. R. Burns, *The Decline of Competition*, New York and London, 1936, page 211: "After the general disruption of prices owing to the war of 1914 to 1918 (*i.e.*, mainly after 1922), the prices of a number of steel products . . . again showed periods of unchanging prices for considerable periods."

(4) The reaction to changes in wages;

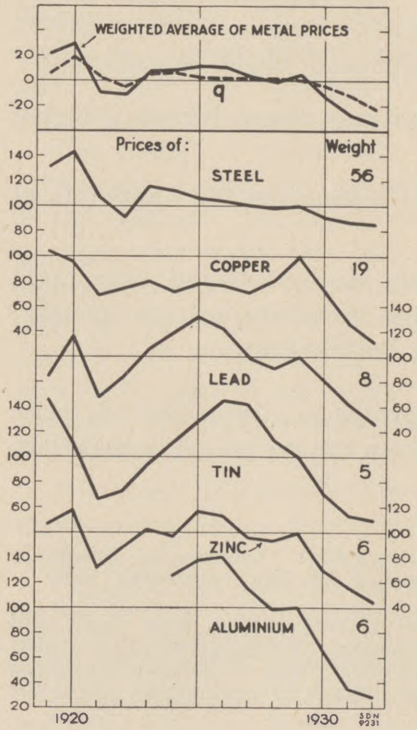
(5) The secular decline in prices as an effect of technical development;

(6) The lag between a change in activity (v') and a change in q. With regard to this lag, however, slightly different assumptions may easily be tried out.

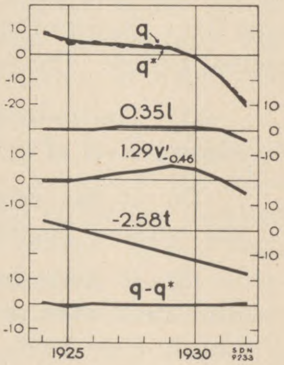
For reasons mentioned under (1) to (3), the correlation calculation has been restricted to the years 1924-1932. An almost perfect correlation was obtained with the formula:

$$q = 0.35l + 0.70v' + 0.59v'_{-1} - 2.58t,$$

Graph 3.51.
PRICES OF METALS.
Indices on the base 1929 = 100.



Graph 3.52.
"Explanation" of
Fluctuations in PRICES
OF INVESTMENT GOODS.



where the coefficient for l had been taken fixed. The two terms with v' may be combined in one, with an average lag of slightly under half a year:

$$q = 0.35l + 1.29\, v'_{-0.46} - 2.58t \text{ (graph 3.52)} \qquad (3.5).$$

The elasticity of supply (with the qualifications given in the general introduction to this chapter)[1] is about 5.

With the help of this formula, the possible differences between this period and the years 1919-1923 may be tested. For this purpose, we compared, in a scatter diagram (3.53)[2] $q - 0.35l + 2.58t$ with $1.29v'$, using different lags of 0.46, 0, and 1 year, in parts II, III and IV. Part I shows the points 1924-1932, which lie nearly on a straight line, at 45°.[3]

If we look at II, we see that 1919, 1921, 1922 are on a straight line with a slope of $3 \div 1$. Considering this line as the regular supply curve for this period, we may deduce that the price was three times as flexible before 1923 as after that year. A conclusion as to a possible difference in level brought about by re-monopolisation may be derived from the point of intersection of the supply line with the q-axis. In I, this point lies at about $q = 1$; in II at $q = 0.5$. This difference is too small to support the evidence that the organisation of the market has had a tendency to raise the level of q.

The points for 1920 and 1923 show a q which is definitely above the supply line. During both years there occurred, as is well-known, a bottle-neck, which was more pronounced in 1920 than in 1923. The two deviations are therefore quite acceptable. It may be seen that the price in 1920 was higher than in 1923, though production (v') was slightly lower. But certainly capacity was higher in 1923, causing bottle-necks to develop only at a

---

[1] *I.e.*, the figure is rather an inverted measure of the flexibility of prices, but it may, in the long run, be an indication of the real elasticity of supply as well.

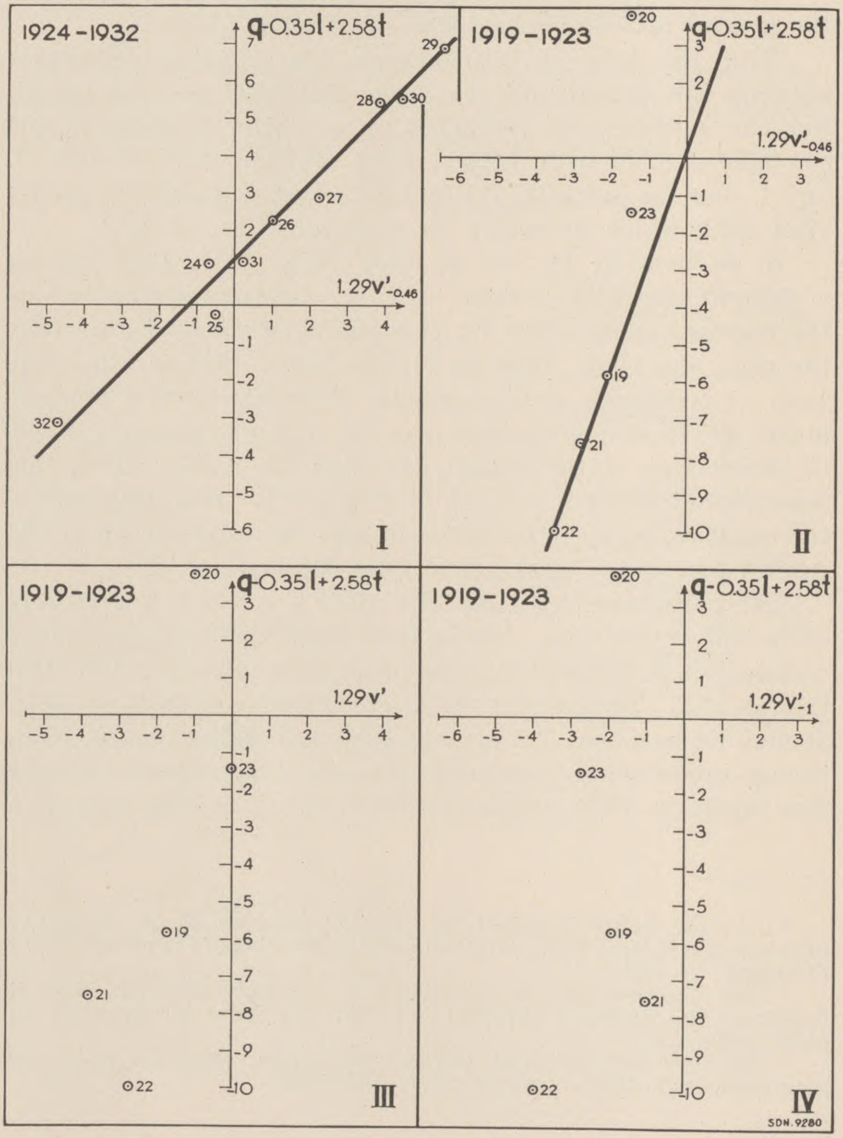
[2] This diagram may be considered as a supply schedule, since it compares the quantity supplied (v') with the price q, corrected for other influences.

[3] The line does not pass through the origin, since the series are measured from their averages over 1919-1932.

Graph 3.53.

PARTIAL SCATTER DIAGRAM
between q (corrected for other influences) and v'.

I 1924-1932, lag 0.46 year
II, III, IV 1919-1923, lag 0.46 ,, (II); 0 ,, (III); 1 ,, (IV)



somewhat larger production. The figures do not show, of course, that these bottle-necks could have been prevented, had the industry been as strongly organised as was the case after 1923. In fact, production has probably never since been so near the limits of capacity.

The alternative cases III and IV are much less satisfactory. In neither case do the points 1919, 1921, 1922 clearly determine the supply line. In IV, moreover, the point 1923 lies to the left of those of 1919 and 1921, which would suggest a bottleneck in the former year at both lower production and — presumably — higher capacity than the latter two. Thus, the material would seem to confirm the assumption made under (6) above.

Equation (3.5) is the "quasi supply relation" for investment goods — a price fixation equation solved for the quantity supplied, where the lag involved may be in contradiction to the Walrasian interpretation of a supply function. It may be written:

$$ {}_{S}v' = 0.78\, q_{+0.46} \tag{3.51}$$

where ${}_{S}v'$ indicates the quantity supplied, corrected for the factors making for shifts of the supply curve:

$$ {}_{S}v'_{-0.46} = v'_{-0.46} + 0.27l - 2.00t \tag{3.52}$$

For the years 1919-1923, the equations are:

$$ {}_{S}v'_{-0.46} = 0.26q \tag{3.51'}$$

and

$$ {}_{S}v'_{-0.46} = v'_{-0.46} + 0.09l - 0.67t. \tag{3.52'}$$

It may be interesting to compare this "quasi supply equation" with the demand equation for investment goods:

$$ v' = 0.66\, Z^{c}_{-\frac{1}{2}} - 0.94\,(m_{Ls})_{-\frac{1}{2}} - 0.03\, q_{-\frac{1}{2}} + 0.12\,(p - \tfrac{1}{2} l)_{-\frac{1}{2}} + 0.63t \qquad (2.4).^{1}$$

---

[1] Replacing 0.33 $(Z^{c} + Z^{c}_{-1})$ by 0.66 $Z^{c}_{-\frac{1}{2}}$, etc.

This equation may be written in a form similar to (3.51):

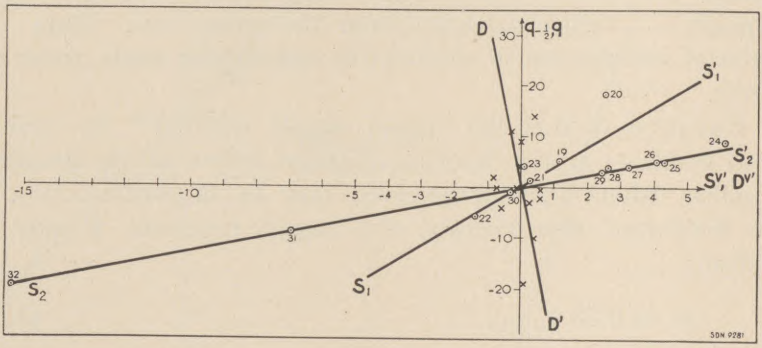
$$_{\mathrm{D}}v' = -0.03q_{-\frac{1}{2}}\text{ [1]} \qquad (3.53)$$

where

$$_{\mathrm{D}}v' = v' - 0.66\,Z^c_{-\frac{1}{2}} + 0.94\,(m_{\mathrm{L}s})_{-\frac{1}{2}} - 0.12(p - \tfrac{1}{2}\,l)_{-\frac{1}{2}} - 0.63t \qquad (3.54).$$

Graph 3.54.

Scatter Diagram of
Demand and Supply Relations for Capital Goods.



Graph (3.54) shows the supply relation (3.51) and the demand relation (3.53) in one diagram. The values for q (with the appropriate lag or lead) are measured along the vertical axis, those for $_{\mathrm{S}}v'$ and $_{\mathrm{D}}v'$ along the horizontal axis. The dots indicate the supply relation (3.51); they show — except for the bottle-neck values of 1920 and 1923 — only small deviations from the two supply curves $S_1S'_1$ for 1919-1923 and $S_2S'_2$ for 1924-1932. The demand relation (3.53) is plotted by little crosses; here the points fit the demand curve DD′ less well. The scale has been chosen in such a way that the 45° lines represent an elasticity of unity.

This diagram illustrates the possibility, in this case, of deriving both the supply relation and the demand relation from one set of price and quantity data, since (i) the lag (lead) between v' and q is different in both relations; and

---

[1] It follows that the elasticity of demand is $(0.03 \times \bar{q}/\bar{v}') = 0.2$.

(ii) the other factors entering into the supply equation are different from those entering into the demand equation.[1]

The demand and supply equations for labour and consumption goods might be analysed in much the same way; the present case has been singled out only because it provides the clearest example.

(3.6) "EXPLANATION" OF BUILDING COSTS

For our purpose, it has not been necessary to give much attention to this equation. In the elimination process, the product of v_B and q_B is the only instance where q_B is used.[2] As the elasticity of demand for v_B is not far from 1,[3] this product is only slightly dependent on q_B; and as, moreover, the absolute value of building is rather small, the dependence of V_B on q_B may be neglected altogether. Hence, for the system of equations as a whole, we do not need to have an equation "explaining" q_B.

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[1] *Cf.* Vol. I, pages 62-64.

[2] *Cf.* Appendix B, Table III, equation (5.10)′.

[3] $0.031 \times \bar{q}_B / v_B$.

CHAPTER IV

DESCRIPTION OF THE MODEL. DEMAND AND SUPPLY IN THE MONEY AND CAPITAL MARKETS

(4.0) Introduction

In this chapter, it is proposed to discuss the price formation of bonds and shares, and some connected problems. This subject belongs to the "Theory of Assets", in which an important development has recently taken place.[1] In this theory the subjects considered are the various holders of assets; and the assets considered are of various types: land, buildings, machines, commodity stocks, securities, short claims and money. Following the principle of this study, we have, in order to make our formulæ manageable, grouped the subjects under three types — *viz.*, banks, other firms, and individuals. Moreover, as physical assets have already been treated separately,[2] we shall be concerned here only with monetary assets.

With regard to the assets which are considered here — viz: bonds, shares, short claims, and money[3] — the simplifying

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[1] J. M. Keynes, *The General Theory of Employment, Interest and Money*, London, 1936; B. Ohlin, "Some Notes on the Stockholm Theory of Savings and Investments", II, *Economic Journal*, 47, June 1937; J. M. Fleming, "The Determination of the Rate of Interest", *Economica* 5, August 1938; H. Markower, and J. Marschak, "Assets, Prices and Monetary Theory", *Economica* 5, August 1938; J. Marschak, "Money and the Theory of Assets", *Econometrica* 6, October 1938.

[2] *Cf.*, on this separation, page 74.

[3] In this chapter, the terms used for monetary assets have the following range:

Bonds: All private and public long-term debt + preferred stock.

Shares: Common stock held by individuals (not by firms).

Short claims: Loans by all banks + Bills discounted and bills bought by the Federal Reserve Banks + Short-term Government debt.

Money: Time + Demand deposits of all banks + Currency held by the public.

(The composition of the series is given in detail in appendix D.)

assumption has been made that each of the three types of subject either demands or supplies each type of asset in the way sketched in the following skeleton table:

	Type of assets			
	Bonds (B)	Shares (C^i)	Short claims (B_s)	Money (M)
Supplied by	Other firms	Other firms	Other firms Individuals	Banks
Demanded by	Individuals (B^i) Banks (B^b)	Individuals	Banks	Other firms Individuals

Banks are supposed to hold only bonds and short claims, the nominal value of which is B^b and B_s respectively; they are the only suppliers of money.

Other (*i.e.*, non-banking) firms supply bonds, shares and short claims, and demand only money; the holding of a considerable part of all shares by these firms seems to be determined rather by the desire for control than by that of earning dividends, and these shares may therefore be altogether eliminated from our collection of assets.

Lastly, individuals exert a demand for bonds and shares (nominal values B^i and C^i respectively) and money; they supply short claims for speculative purposes.

To these simplifications we may further add the assumption that the holding of money is independent of the holding of bonds or shares. The reasons for this are the following:

(1) A considerable part of total money is held by "other firms", which we assumed to hold no securities for investment purposes.

(2) A large class of individuals who hold money are not in the position to hold shares or bonds.

In all other cases where different types of assets are supplied or demanded by one group of subjects, the supply (demand) of various types has been studied jointly. Thus, the supply (demand) of each type of assets has been taken to be dependent on the total supply (total demand) and the prices of all types of assets supplied (demanded) by the group of subjects.

On the other hand, the demand and supply decisions of one group of economic subjects are considered as *independent*. Here a parallel may be drawn with the separate maximisation that is often supposed to exist for the individual's way of earning and spending income: *first* he trives to get the maximum money income, and *then* he seeks the maximum satisfaction from the given amount of money. Likewise, we may assume that firms decide *first* what is necessary for the course of production (the construction of buildings and machines, the holding of commodity stocks and the amount of debt and shares they are prepared to carry), and *afterwards* how much money they need to keep to these plans. Banks *first* decide how much money they will allow to be in existence, and *then* distribute this amount over short claims and bonds. Speculators *first* determine their holdings, and *then*, if necessary, borrow short credits.

It also follows from this division that buildings, machines and commodity stocks do not enter into consideration in this chapter. Their creation and prices have been treated separately in previous chapters.

Summarising, we may divide our task, as set out in the skeleton table on the preceding page, into five parts:

(i) The joint supply of bonds, shares and short claims by other firms and, with regard to the last item, speculating individuals (sections (4.1) to (4.3));

(ii) The supply of money by the banks (section (4.4));

(iii) The joint demand for short claims and bonds by the banks (section (4.5));

(iv) The demand for money by other firms and individuals (section (4.6)).[1]

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[1] For reasons that will be explained in section (4.7) it is necessary to treat the demand for money before the demand for bonds and shares by individuals.

(v) The joint demand for bonds and shares by individuals (sections (4.7), (4.8), (4.9)).

(4.1) THE SUPPLY OF BONDS

The total amount of bonds outstanding at any moment (B_0) may be considered as the sum of the amounts outstanding a year before (B_{-1}) + the increase over that year (ΔB). To "explain" the supply of bonds, it seems statistically most expedient first to "explain" ΔB and then to cumulate the equation found.[1]

It may be remarked that the "explanations" given in this section and the next are rather rough because (i) the material is not good enough to allow of very much refinement and (ii), as will be shown below,[2] we shall, in any case, be obliged to approximate the "explanation" found by a mere trend term.

In view of the difference in determining factors, ΔB has been split into:

ΔB^e: bonds issued by private enterprise, States and local governments;[3]

ΔB^g: bonds issued by the Federal Government.

(4.11) ΔB^e

I. *Theoretical.*

The chief determining factors of changes in the amount of these bonds (and, equally, of shares) outstanding are assumed to be: (i) changes in the value of the stock of capital goods, and (ii) the rates of interest which determine on which market

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[1] Issue figures could not be used instead of ΔB, since, though they are in themselves more certain than the B-figures, they represent only a part of the fluctuations in B. They have, moreover, the disadvantage that they do not cover capital reductions.

[2] *Cf.* section (4.7).

[3] The fluctuations in the increase in debt of States and local governments are too small to justify special treatment. It seemed most logical to combine this debt with that of private enterprise, with which it has in common the important factor of a limited market for its issues. Hence State and local issues fell abruptly after 1930, at the same time as the federal debt heavily increased.

the new capital goods will be financed. In greater detail, this leads to the consideration of the following series:

(1) The value of investment goods delivered (V'). It is possible that there is usually a lag between the production of investment goods and the final financing with long-term capital. Hence V'_{-1} may be included next to V'.[1]

(2) The value of depreciation, as reflected in regular repayments. These repayments have been considered as a constant.

(3) The value of writings-off, as reflected by capital reductions (this factor is probably more important for shares than for bonds). Writings-off may be considered as a readjustment of the value of the capital on a replacement basis; they may therefore be represented by the rate of change in the price of capital goods, Δq. Since writings-up are unusual, only the negative values of Δq should be taken into account; this truncated series may be represented by $(-\Delta q)''$, the sign $''$ indicating that only positive values of the expression between brackets are taken into account.

(4) m_{Lb} and m_{Ls}, the interest rates on the bond and the share market. In the "explanation" of the supply of bonds, the first series may be expected to have a negative coefficient and the second a positive; and inversely in the "explanation" of the supply of shares.[2]

(5) The alternative to issuing bonds or shares consists in (temporarily) financing with short-term credit. The price of

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[1] In principle, series for stocks like B and C refer to the average of the year. Consequently, if ΔB and ΔC are to represent the increase during a calendar year, they should be calculated as the difference $B_{+\frac{1}{2}} - B_{-\frac{1}{2}}$, etc., and not as $B - B_{-1}$. For B^g and C, this has actually been done. The series B^e, however, is not accurate enough to be placed at any precise date. Hence, $B - B_{-1}$ has been taken to represent ΔB. It follows that the lag found for this series should be very carefully interpreted.

[2] A parallel may, however, be drawn here with the signs to be expected for the price coefficients of two goods on which a very large part of income is spent (*cf.* section (2.1), page 43). Hence, since a very large part of all investments is financed either by bonds or by shares, the signs for m_{Lb} and m_{Ls} may be different from those to be expected according to the general rules for commodities on which only a small part of income is spent.

this credit (m_S) has not been included in these equations, since its influence must be of minor importance as compared with that of m_{Lb} and m_{Ls} (*cf.* section (4.3)).

II. *Statistical.*

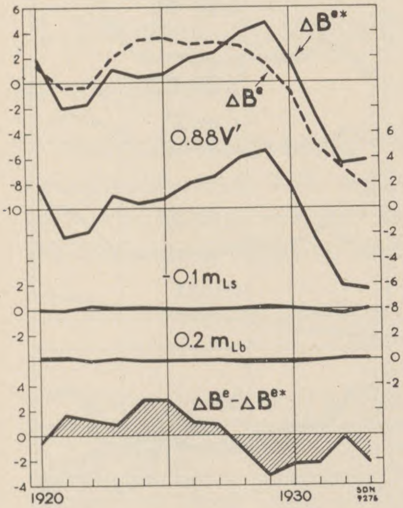
In the "explanation" of ΔB^e by the five series mentioned, a negative coefficient was found for V'_{-1}, which pointed to a lead of some months. Since this does not seem acceptable, and since a fixed combination of V' and V'_{-1}, representing a lag of half a year, gave a much worse correlation, a case without V'_{-1} was finally chosen. Here the coefficient for $(-\Delta q)''$ was so small that it was left out.

The formula finally accepted runs:

$$\Delta B^e = 0.88V' - 0.1m_{Ls} + 0.2m_{Lb} \qquad (4.11).$$

Graph 4.11.

"Explanation" of Fluctuations in the INCREASE OF PRIVATE LONG-TERM DEBT OUTSTANDING.



The signs for m_{Lb} and m_{Ls} are not in accordance with theoretical expectation (in its simplest form); but the coefficients do not seem to be very significant, and the influence of both series is very small (*cf.* graph 4.11).

(4.12) $\underline{\Delta B^g}$

The total increase in debt of the Federal Government, $\Delta B^g + \Delta B^g_s$,[1] is, by definition, equal to the Government's expenditure minus its revenue. Federal expenditure — in distinction to that of the lower authorities — rises in depressions, when relief payments of different kinds have to be paid, and falls in years of prosperity; revenue,

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[1] *I.e.*, the increase in long-term and short-term Government debt.

depending on incomes, imports and similar items, tends to behave in the opposite way. As a result of both causes, the increase in debt will move counter to the cycle. On the basis of these considerations, an attempt is made to " explain" $(\Delta B^g + \Delta B^g_s)$ by Z^c and Z^c_{-1} — in order to take account of possible lags — and a trend to represent a possible second-degree trend in Government debt.[1] Over the period 1920-1933, a very satisfactory fit was found with the following formula:

$$\Delta B^g + \Delta B^g_s = -0.115 Z^c - 0.155\, Z^c_{-1} + 0.138 t \qquad (4.121).$$

In order to find ΔB^g, the short-term debt may be explained separately. It stands to reason that here considerations with regard to the rate of interest are most important, in such a way that the *amount* of short-term debt outstanding (and not its *increase*) depends positively on the long-term rate of interest.[2] This hypothesis is fairly well confirmed by the facts:

$$B^g_s = 1.7\, m_{Lb} \qquad (4.122).$$

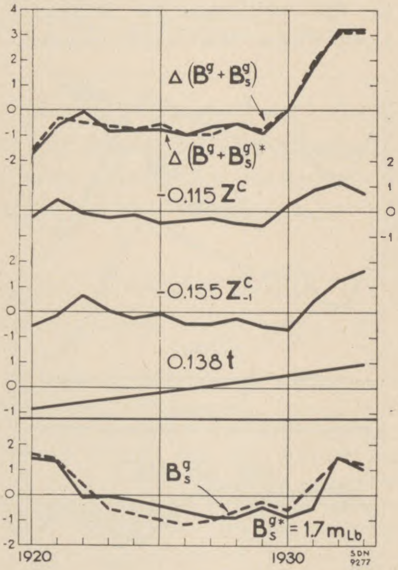
It follows that ΔB^g depends on Z^c, Z^c_{-1}, t and the rate of increase in m_{Lb}:

$$\Delta B^g = -0.115\, Z^c - 0.155\, Z^c_{-1} - \\ -1.7\, \dot{m}_{Lb} + 0.138 t \quad (4.123).$$

Graph 4.12.

" Explanation " of Fluctuations in the INCREASE OF FEDERAL GOVERNMENT DEBT.

" Explanation " of Fluctuations in SHORT-TERM FEDERAL GOVERNMENT DEBT OUTSTANDING.



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[1] A stable increase (linear trend) in Government debt would mean a constant Δdebt; a linear trend in Δdebt represents a second-degree trend in debt.

[2] The short-term rate of interest, being of minor importance, has not been included.

(4.13) *Cumulation.*

The series B may be found by cumulating $\Delta B^e + \Delta B^g$:

$$B = 0.88\int V' - 0.1\int m_{Ls} + 0.2\int m_{Lb} - 0.115\int Z^c - 0.155\int Z^c_{-1} - 1.7 m_{Lb} + 0.07t^2 + 4.3t \qquad (4.131).$$[1]

Cumulations over the period covered come very near to a trend. Hence B may fairly well be approximated by a trend:[2]

$$B = 4.88t \qquad (R = 0.99) \qquad (4.1).$$

It will be taken in this form in section (4.7).

(4.2) THE SUPPLY OF SHARES

(4.21)

I. *Theoretical. Cf.* section (4.11).

II. *Statistical.*

Here, as with bonds, V' had a tendency to show a considerable lead with regard to ΔC, instead of the expected lag.[3] Hence V'_{-1} was also omitted, and so was $(-\Delta q)''$, which showed a small negative coefficient. The equation chosen runs:

$$\Delta C = 1.64\, V' - 1.1\, m_{Ls} + 8.8\, m_{Lb} \qquad (4.21).$$

---

[1] The coefficient for t is equal to the average of ΔB; that for t^2 is found by integrating $0.138t$ from (4.123).

[2] The m_{Lb}-term in (4.131), originating with that in (4.122), which is virtually not a cumulant, is small. Moreover, in all correlation calculations where t will be used as representing B, m_{Lb} will also be included as a separate variable.

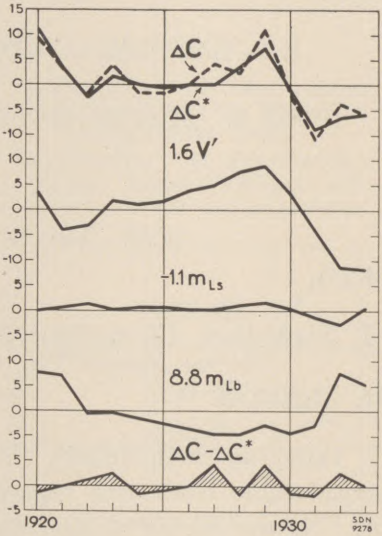
The second-degree trend has a very small influence.

[3] The particular conditions of the country and period under review may perhaps explain why the figures show this lead. In the years 1927 to 1929, part of the receipts of share issues were used on the stock exchange, and were only later taken up by investment.

The signs for m_{Ls} and m_{Lb} are as expected. The coefficient for V′ is very large. Adding that for V′ in (4.11), we find that the fluctuation in the total net increase in long-term capital is about two and a half times as large as the fluctuation in investment. This does not seem reasonable; probably V′ to a certain extent takes the place of another variable not included. In view of the use that will be made of these equations, it is not at present necessary to go deeper into this question.

Graph. 4.21.

"Explanation" of Fluctuation in the INCREASE OF SHARE CAPITAL OUTSTANDING.



(4.22) *Cumulation.*

Cumulating (4.21), we get:

$$C = 1.64\int V' - 1.1\int m_{Ls} + 8.8\int m_{Lb} + 2.4t \qquad (4.22),$$

which is again simplified to:

$$C = 3.18\,t \quad (R = 0.95) \qquad (4.2).$$

(4.3) THE SUPPLY OF SHORT CLAIMS

The supply of *short-term claims* reacts, by the very nature of these claims, much more quickly to the economic situation than the supply of stocks and bonds. With only a small margin of error, it may therefore be maintained that this supply depends on the variables to be discussed, without any lag. The supply of claims being synonymous with the demand for loans, we have to consider what factors determine this demand. Three seem to be outstanding:

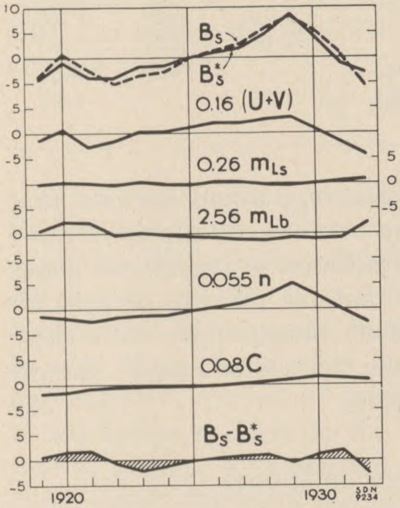
(i) Short-term interest rates (m_S);

(ii) Total value of all shares $0.0156\,\overline{\overline{n}}\,\overline{\overline{C}}$ (*cf.* relation 1.1) representing a demand factor for loans for speculative purposes;

(iii) Total value of production (U+V), representing a demand factor for industrial and commercial loans. This demand is composed of two parts: viz., a demand corresponding to working capital and a demand corresponding to the provisional financing of new investments, which is usually consolidated only after a certain time. The former shows fluctuations which are fairly accurately parallel to U+V, as is seen by inventory statistics, whereas the latter will be parallel to V, as only the new investment of some short period immediately before is financed in this way. Given a high parallelism between U and V and the rather subordinate rôle the present relation is found to play in the whole system, no attempt has been made to distinguish between the influence of U and that of V.

Graph 4.3.

“Explanation” of Fluctuations in SHORT CLAIMS BY SUPPLY FACTORS.



(iv) As competitive “prices”, share yield (m_{Ls}) and bond yield (m_{Lb}) may be added.

Using these explanatory variables we find the following regression equation:

$$Bs = 0.16\,(U+V) + 0.26m_{Ls} + 2.56m_{Lb} + 0.055n + 0.08C \qquad (4.3).$$

The variable m_S has been omitted, since it obtained an insignificant positive coefficient, which is not in accordance with theoretical expectations. The value 0 which we have chosen would mean that the elasticity of this supply (which, as has been said already, is equivalent to the demand for short loans) would be *zero.* This is in harmony with our results concerning the low influence of interest rates on investment activity, as well as our hypothesis about the small influence of short-term rates on the share market.

Summary of Results of Correlation Calculations concerning the Supply of Short Claims.

Case	Regression coefficient for:					R
	U + V	C′	m_{Ls}	m_{Lb}	m_S	
1	0.11	0.04	2.27	− 3.93	1.89	0.98
2	0.18	0.05		1.40	− 0.07	0.96
3	0.19	0.05	1.22		0.67	0.98
4	0.21	0.05	0.63	1.25		0.97

The influences found for m_{Ls} and m_{Lb} would indicate that there is a considerable competition between the supply of short claims on the one hand, and the flotation of shares and bonds on the other hand. It is natural that the price to be paid for long-term credits should have much more weight with those who demand these credits than the short-term rate of interest which is to be paid only temporarily.

(4.4) THE SUPPLY OF MONEY

The supply of money may be split up into:

(4.41) the supply of currency, and
(4.42) the supply of deposits.

Indicating total money by M, currency outside banks by M′ and deposits by M″, we have

$$M = M' + M'' \qquad (4.40).$$

(4.41). The item "Money in circulation" or "Currency in circulation" of the Federal Reserve Banks balance-sheet covers not only M′, but also the currency held by banks: vault cash (VC). It may be assumed that, for both items, supply follows demand automatically;[1] hence for both we have to consider demand only (*cf.* section (4.6)).

(4.42) In principle, the supply of deposits may be said to be regulated by acts of price fixation — *i.e.*, fixation of the short-term interest rate m_S — by the commercial banks, on the basis of their debt position with the Federal Reserve Banks. The fact that debts (in the form of rediscounts) are permitted to be incurred only for a short period creates a tendency for the banks to fix their interest rates in such a way as to avoid such debts.[2] This means that the higher the net debt position, indicated by bills rediscounted (Bi) minus excess reserves (R^e), the higher the rate fixed.[3] This may be indicated by a relation:

$$m_S = f\,(Bi - R^e) \qquad (4.420).$$

This relation is shown in graph 4.421. Monthly figures for $Bi - R^e$ (abscissa) are plotted against m_S (ordinate). Where space has allowed, the different months of one year have been connected, and the first and last months indicated. Yearly figures have been plotted on 4.422, with the same scale. It will be seen that, on the right-hand part of the graph, where Bi outweighs R^e, the rate of interest rises steeply with an increase in indebtedness. More to the left, however, the reaction of the rate of interest to a position of large excess reserves becomes ever fainter; evidently m_S cannot be lower than 0, or a trifle above 0.

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[1] *Cf.* L. Currie, *The Supply and Control of Money in the United States*, Cambridge (Mass.), 1935, Chapter X.

[2] *Cf.* W. W. Riefler, *Money Rates and Money Markets in the United States*, New York and London, 1930.

[3] One might, moreover, have expected to find an influence exercised by the gold stock — viz., a raising of discount rates when the gold cover of the liabilities of the Federal Reserve Banks becomes low. However, no evidence of such an influence is found.

The period considered in this study shows points that are nearly all on the right-hand side of the graph. For this part of the diagram, we may approximate the function of (4.420) by a straight line:

$$m_S = 4\,(Bi - R^e), \qquad (4.421)$$

where the factor 4 indicates that the banks raise their rate of interest by 1 % when their indebtedness with the Federal Reserve Banks increases by $250 million. It will be seen that this line very well fits the scatter in the years 1919-1932.

The original figures on indebtedness, as published by the Federal Reserve Banks, did not follow this pattern for the years 1917 to 1921 (*cf.* the black dots on graph 4.422). In these years, large amounts of United States war paper had to be absorbed by the banking system; this could only take place at the cost of increased indebtedness with the central banks. The special causes that were at work during these years made it reasonable not to include this indebtedness in B*i*,[1] but rather to regard it as Federal Reserve Banks holdings of Government paper.

Accordingly, all B*i* figures for 1917 to 1921 have been diminished by the amount of bills secured by Government paper each month *minus* the average amount so secured in 1922 ($230 million) — a year when normal conditions had presumably been restored.

The value of $Bi - R^e$ itself is determined by the other items occurring in the combined balance-sheets of the Federal Reserve Banks, which may be summarised as follows:

Liabilities		*Assets*	
Member bank reserve balances required	R^r	Gold stock	A*u*
Member bank reserve balances, excess	R^e	Bills discounted . .	B*i*
Currency in circulation	$M' + VC$	Bills bought, Government securities and all other items	P

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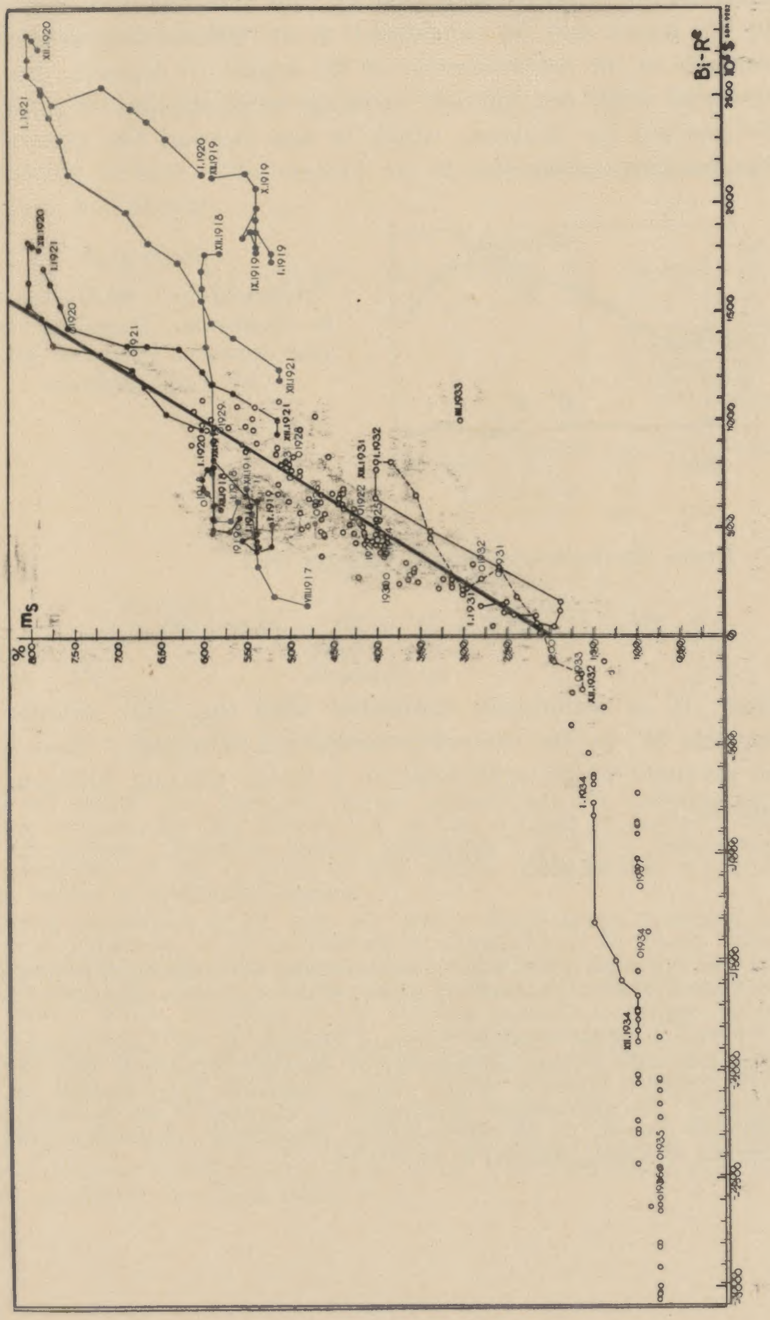
[1] *Cf.* RIEFLER, *loc. cit.*, page 158.

Graph 4.422.

Relation between the Banks' Indebtedness with the Federal Reserve Banks ($B_i - R^e$) and the Short-term Rate of Interest (m_S).

Monthly figures, 1917-1921, 1931-1937 (•••) and yearly figures, 1919-1937 (∘∘∘) (1919-1921 corrected).

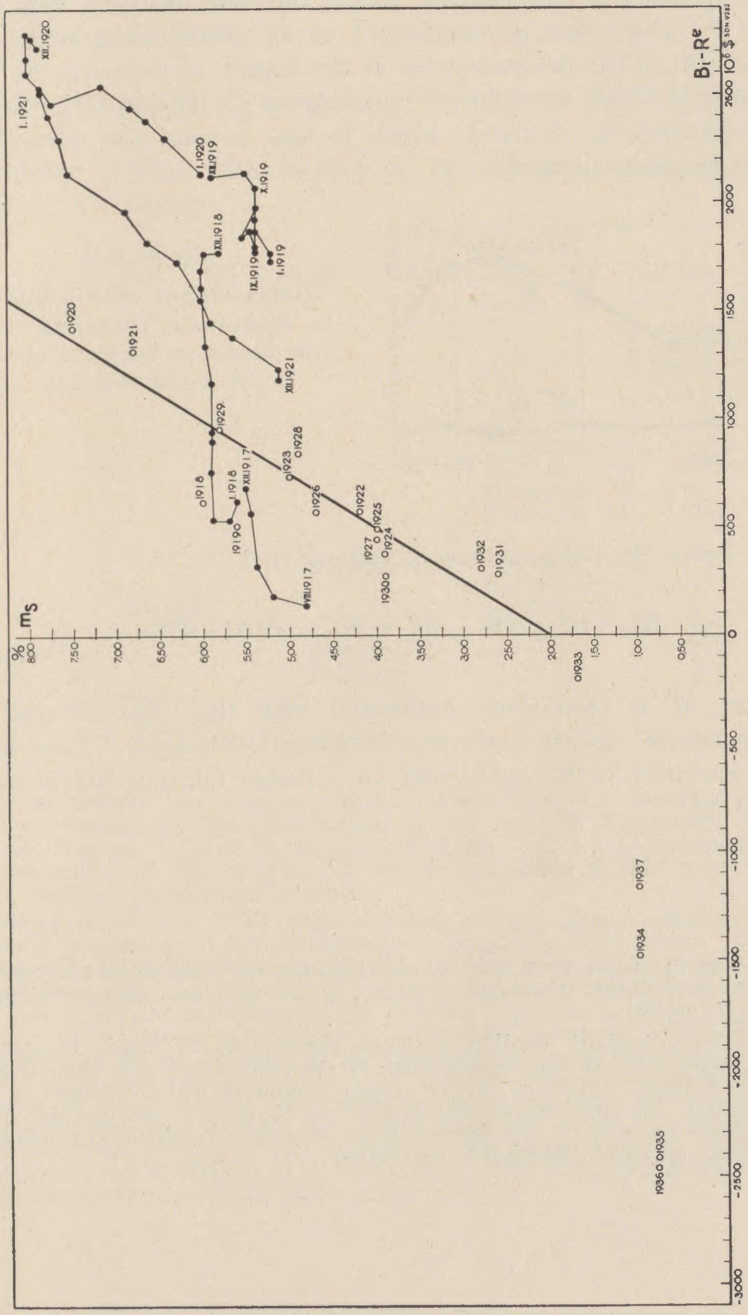
Monthly figures, 1917-1937 (1917-1921 corrected).



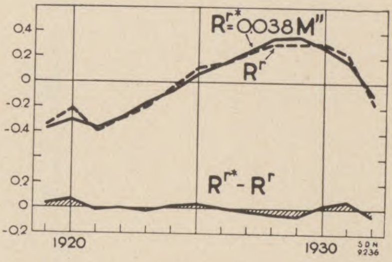
Graph 4.422.

Relation between the BANKS' INDEBTEDNESS WITH THE FEDERAL RESERVE BANKS ($Bi - R^e$) and the SHORT-TERM RATE OF INTEREST (m_S).

Monthly figures, 1917-1921, uncorrected (••••) and yearly figures, 1919-1937 (∘∘∘∘) (1919-1921 corrected).



The last item includes all small assets minus all small liabilities not elsewhere mentioned;[1] its major constituents, however, are bills bought and Government securities. As these are the chief instruments of open-market policy, the item has been indicated by P, which will be considered as an autonomous (external) variable in the determination of the supply of deposits. So will the gold stock A*u*, whereas currency in circulation (M′ + VC) is determined by demand, which is also beyond the control of the banking authorities.



Graph 4.43.
"Explanation" of Fluctuations in REQUIRED RESERVES WITH THE FEDERAL RESERVE BANKS by Total Deposits.

From the balance-sheet it follows that

$$Bi - R^e = R^r + M' + VC - (Au + P).$$

Here, R^r is technically connected with the total amount of deposits M″ by the reserve prescriptions. Roughly,[2] these may be assumed to be equivalent to a linear relation between R^r and M″:

$$R^r = \mu M''.$$

---

[1] In the years 1919-1921, P also includes the amount of rediscounts on United States Government paper, which has been subtracted from B*i* (*cf. supra*).

[2] The constant relation between changes in M″ and changes in R^r exists only if the composition of deposits and their distribution over different types of banks change regularly with changes in all deposits. The percentage distribution of changes in all deposits over different groups, as determined from correlation calculations (taking account of trend changes) is as follows:

It follows that the price fixation equation for m_S may be written as

$$m_S = 4\ [\mu\, M'' + M' + VC - (Au + P)] \qquad (4.422)$$

or, according to (4.40)

$$m_S = 4\ [\mu\, M + (1 - \mu)\, M' + VC - (Au + P)] \qquad (4.423).$$

Solving (4.423) with respect to M, we find as a supply equation for money:

$$M = \frac{m_S}{4\mu} - \frac{1}{\mu}\, [(1 - \mu)\, M' + VC - (Au + P)] \qquad (4.424).$$

				Required reserves with Federal Reserve Banks 1917-1936
All banks: 100	Member banks: 72	Demand: 39	Central reserve cities: 9	13%
			Reserve cities: 13	10%
			Country: 13	7%
			Other (Government etc.): 4	0%
		Time: 33		3%
	Non-member banks: 28	Demand: 10		0%
		Time: 18		0%

If we weight the required reserve percentages, as indicated in the last column, by this distribution of the changes in deposits, we find an average "marginal reserve percentage" of 4.3 — indicating the possibility of the creation of \$23 million of additional deposits on \$1 million of additional reserves.

Direct correlation of R^r with M'' and a trend gives, however, a marginal percentage of 3.8 (indicating an expansion of 26 times). The difference between the two figures is probably due to the fact that, in a depression, idle money with the country banks is redeposited with city banks; which, according to the existing regulations, obliges both banks to keep reserves against them. In times of prosperity, this money is either used in the country, or directly deposited with the New York banks (1929). For this reason the ratio between reserves required and deposits in the hands of the public tends to be lower in times of prosperity, when M'' is high, and higher when M'' is low (*cf. Member Bank Reserves*, Report of the Committee on Bank Reserves of the Federal Reserve System (1931), pages 9-10). The coefficient found by direct correlation has been taken as μ.

As it has been found by correlation that

$$\mu = 0.038 \text{ (\textit{cf.} graph 4.43)},$$

the following equation is finally taken for the supply of M:

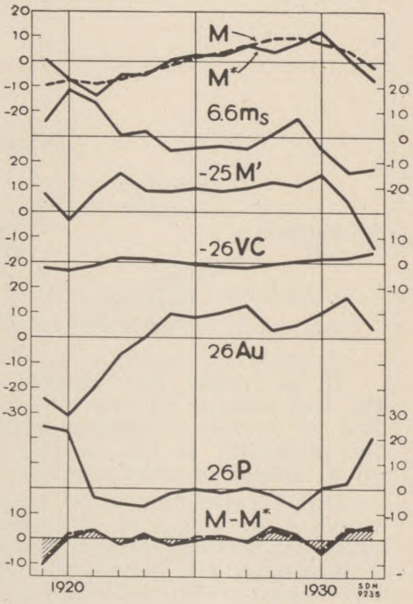
$$M = 6.6\, m_S - 25\, M' - 26\, VC + 26\, (Au + P) \qquad (4.4).$$

Graph 4.4 shows the fit of this relation.[1] It will be seen that M′ has a large *negative* influence on M; the hoarding of some 1 or 2 milliard dollars of currency in 1931 and 1932 must, in particular, have caused 25 times as large a decrease in the supply of money.

Graph 4.4.

"Explanation" of Fluctuations in the QUANTITY OF MONEY by Supply Factors.

(In the residuals, a dotted line is drawn indicating 26 × the residuals of the yearly figures in graph 4.422.)



(4.5) Demand by Banks for Short Claims and Bonds

As has already been stated, the demand by banks for short claims and for bonds is considered as joint. This means that the total amount which the banks have available to hold assets is distributed over the two categories of assets in a way depending on the price and the attractiveness of each. This amount has been derived from the combined balance-

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[1] The fit is not very good. This is due to the fact that (4.4) is found by solving (4.423) with respect to M, which plays a minor rôle in this equation. Hence the residuals have a larger relative importance. Comparison of the residuals with 26 times the residuals in (4.422) shows that the former are almost entirely due to the latter.

sheets of all banks, including the Federal Reserve Banks, which for this purpose may be summarised as follows.[1]

Liabilities			*Assets*	
Outside currency	$\overline{\overline{\mathrm{M}}}'$	$\left.\begin{array}{l} \\ \\ \end{array}\right\} \overline{\overline{\mathrm{M}}}$	Gold stock . . .	$\overline{\overline{\mathrm{A}u}}$
Deposits	$\overline{\overline{\mathrm{M}}}''$		Short claims . .	$\overline{\overline{\mathrm{B}}}_s$
			Bonds	$\overline{\overline{\mathrm{B}}}^b . \frac{c\,*}{\overline{\overline{m}}_{\mathrm{L}b}}$

* For explanation of this term, see below.

All other items are either almost constant or unimportant. The amount available for distribution over B_s and B_b may therefore be taken as equal to $\mathrm{M} - \mathrm{A}u$, or, since the fluctuations in $\mathrm{A}u$ are very small compared with those in M, as equal to M (*cf.* graph 1.6).[2]

The factors determining how this total holding is to be distributed over the two types of assets may be separated into two groups: their prices, and the attractiveness which each asset is expected to have for the holder. The price of short claims is taken as 1. The price of bonds is equal to $\frac{c}{\overline{\overline{m}}_{\mathrm{L}b}}$, where c is the nominal yield (averaged over all bonds in existence) and $m_{\mathrm{L}b}$ the actual yield. The variations in time of c may be disregarded; it will be taken as a constant with the value 4.5; hence bond prices may be taken to vary inversely with $\overline{\overline{m}}_{\mathrm{L}b}$, and, instead of prices, $m_{\mathrm{L}b}$ may be taken as a variable.

The attractiveness of short claims consists in the interest income they yield; m_{S} must therefore be included as an "explanatory" variable. Bonds are in the first place attractive on account of the regular income in the form of interest (c) which they yield; but as c is considered as a constant, it need not be included. A second attraction of bonds may consist in

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[1] The item "vault cash" cancels out.

[2] An attempt was made to include $\mathrm{A}u$ in the explanation of B_s and B^b, but no perceptible influence could be found.

expected price gains at the moment of selling; these gains are supposed to be inversely connected with the rate of increase in $m_{\mathrm{L}b}$: $\dot{m}_{\mathrm{L}b}$.

The banks' demand for short claims will thus be assumed to be a function $\mathrm{D}_1(\mathrm{M}, m_{\mathrm{S}}, m_{\mathrm{L}b}, \dot{m}_{\mathrm{L}b})$ of M, m_{S}, $m_{\mathrm{L}b}$ and $\dot{m}_{\mathrm{L}b}$, and their demand for bonds to be a function $\mathrm{D}_2(\mathrm{M}, m_{\mathrm{S}}, m_{\mathrm{L}b}, \dot{m}_{\mathrm{L}b})$ of the same variables. These functions must be of such a nature that at any moment the total money value of short claims and of bonds held by the banks equals the value M. Since the price of bonds equals $\frac{c}{m_{\mathrm{L}b}}$ and that of short claims is unity, the money value of assets held is $\bar{\bar{\mathrm{B}}}_s + \bar{\bar{\mathrm{B}}}^b \frac{c}{m_{\mathrm{L}b}}$, where $\bar{\bar{\mathrm{B}}}_s$ and $\bar{\bar{\mathrm{B}}}^b$ represent the nominal amounts held in absolute values (and not their deviations from average). This money value must be identically equal to $\bar{\bar{\mathrm{M}}}$ — *i.e.*, equal to $\bar{\bar{\mathrm{M}}}$ for any values of m_{S}, $m_{\mathrm{L}b}$ and $\dot{m}_{\mathrm{L}b}$:[1]

$$\bar{\bar{\mathrm{B}}}_s + \bar{\bar{\mathrm{B}}}^b \frac{c}{m_{\mathrm{L}b}} \equiv \bar{\bar{\mathrm{M}}} \qquad (4.51).$$

---

[1] The treatment chosen here can perhaps best be understood by analogy with the demand for n types of consumers' goods on which together all the income of a certain group of persons is spent. (This presupposes that all consumers' goods are included and that either no saving occurs or saving is also considered as a consumers' good). Denoting the quantities demanded of the various goods by u_1, u_2, u_3, etc., their prices by p_1, p_2, p_3, etc., and total income by Y, the demand functions are:

$$u_1\ (p_1,\ p_2,\ p_3\ \ldots\ p_n,\ \mathrm{Y})$$
$$u_2\ (p_1,\ p_2,\ p_3\ \ldots\ p_n,\ \mathrm{Y}),\ \text{etc.}$$

They will be dependent, since they must fulfil the following relation:

$$u_1p_1+u_2p_2+\ \ldots\ u_np_n = \mathrm{Y}.$$

In our case, assets take the place of consumers' goods and Y is replaced by the value of all assets. In addition, the demand functions depend on other variables, since, unlike consumers' goods, these assets have changing properties which make them in a changing degree attractive to holders.

In deviations from average, this identity may be written as [1]

$$\mathrm{B}_s + 0.9\,\mathrm{B}^b - 2.93\,m_{\mathrm{L}b} \equiv \mathrm{M} \text{ (cf. graph 1.6)} \qquad (4.52).$$

The identity implies that the two demand functions are not independent of each other. Assuming them to be linear, and of the form:

$$\mathrm{B}_s = \Delta_{11}\,\mathrm{M} + \Delta_{12}\,m_{\mathrm{S}} + \Delta_{13}\,m_{\mathrm{L}b} + \Delta_{14}\,\dot{m}_{\mathrm{L}b} \qquad (4.53)$$

$$\mathrm{B}^b = \Delta_{21}\,\mathrm{M} + \Delta_{22}\,m_{\mathrm{S}} + \Delta_{23}\,m_{\mathrm{L}b} + \Delta_{24}\,\dot{m}_{\mathrm{L}b} \qquad (4.54)$$

the coefficients must fulfil certain conditions to guarantee the identity (4.52). It follows that:

$$\left.\begin{array}{ll} \Delta_{11} + 0.9\Delta_{21} & = 1 \\ \Delta_{12} + 0.9\Delta_{22} & = 0 \\ \Delta_{13} + 0.9\Delta_{23} - 2.93 & = 0 \\ \Delta_{14} + 0.9\Delta_{24} & = 0 \end{array}\right\} \qquad (4.55).$$

The correlation calculation to find these coefficients has been made in such a way that these conditions are automatically fulfilled.[2]

---

[1] The calculation runs as follows:

(1) $\bar{\bar{\mathrm{B}}}_s = \bar{\mathrm{B}}_s + \mathrm{B}_s$

(2) $\bar{\bar{\mathrm{B}}}^b = \bar{\mathrm{B}}^b + \mathrm{B}^b$

(3) $\bar{\bar{\mathrm{B}}}^b \dfrac{c}{\bar{\bar{m}}_{\mathrm{L}b}} = (\bar{\mathrm{B}}^b + \mathrm{B}^b) \dfrac{4.5}{\bar{m}_{\mathrm{L}b} + m_{\mathrm{L}b}} = (\bar{\mathrm{B}}^b + \mathrm{B}^b) \dfrac{4.5}{\bar{m}_{\mathrm{L}b}} (1 - \dfrac{m_{\mathrm{L}b}}{\bar{m}_{\mathrm{L}b}})$

(approximately) $= 0.9(\bar{\mathrm{B}}^b + \mathrm{B}^b - 3.26 m_{\mathrm{L}b})$ (neglecting a second order term and using $\bar{m}_{\mathrm{L}b} = 5$, $\bar{\mathrm{B}}^b = 16.3$).

(4) $\bar{\mathrm{B}}_s + 0.9\,\bar{\mathrm{B}}^b = \bar{\mathrm{M}}$.

[2] Instead of requiring separately that:

$\Sigma\,(\mathrm{B}_s - \mathrm{B}_s')^2$ and $\Sigma\,(\mathrm{B}^b - \mathrm{B}^{b\prime})^2$ be a minimum,

we require that $\Sigma\,(\mathrm{B}_s - \mathrm{B}_s')^2 + \Sigma\,(\mathrm{B}^b - \mathrm{B}^{b\prime})^2$ be a minimum.

In this function B_s' and $\mathrm{B}^{b\prime}$ are replaced by (4.53) and (4.54), and four of the eight coefficients are eliminated with the help of (4.55). From the function so obtained, four normal equations are derived in the ordinary way. (*Cf.* Vol. I, pages 133-136.)

The numerical results found are

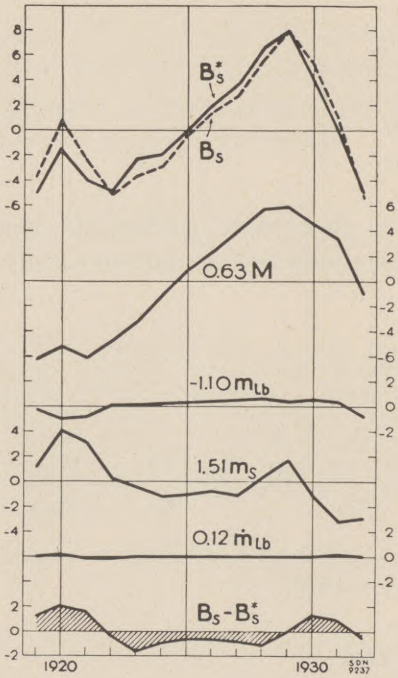
$$B_s = 0.63\ M + 1.51\ m_S - 1.10\ m_{Lb} + 0.12\ \dot{m}_{Lb} \qquad (4.56)$$

$$B^b = 0.41\ M - 1.68\ m_S + 4.48\ m_{Lb} - 0.14\ \dot{m}_{Lb} \qquad (4.57).$$

The fits are good, as is shown by graphs 4.56 and 4.57; the influence of $\dot{m}_{Lb}$ is negligible.

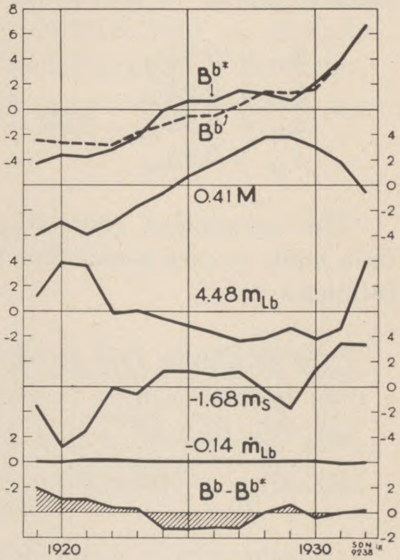
Graph 4.56.

"Explanation" of Fluctuations in the VALUE OF SHORT CLAIMS by Demand Factors.



Graph 4.57.

"Explanation" of Fluctuations in the NOMINAL VALUE OF BONDS HELD BY THE BANKS, by Demand Factors.



(4.6). THE DEMAND FOR MONEY

The demand for money may be split into:

(4.61) Demand for currency by the public ("outside currency");

(4.62) Demand for currency by the banks,[1]

(4.63) Demand for deposits.

(4.61). M′. The demand for “ outside currency ” [2] consists of two parts:

(*a*) demand for payments to and by workers and farm population, which may be taken as linearly dependent on total wages and salaries $L_w + L_s$, plus agricultural money income E'_F; and

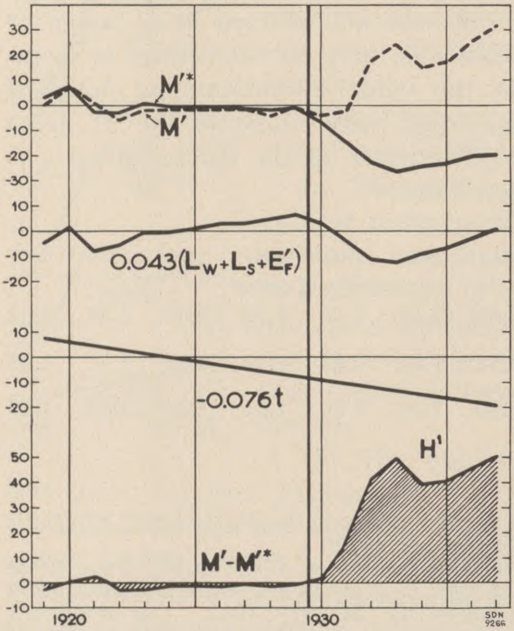
(*b*) demand for idle money (hoards).

The left-hand side of graph 4.611 shows that for the years 1919-1929, when it may be taken that there was no considerable currency hoarding, the course of M′ may be very well explained by the movement of $L_w + L_s + E'_F$ and a negative trend, indicating the increasing use of cheques instead of currency. The formula runs:

$$M' = 0.043\,(L_w + L_s + E'_F) - 0.076\,t \qquad (4.611).$$

Graph 4.611.

“ Explanation ” of the Fluctuations of Outside Currency by Demand Factors, 1919-1929. *Extrapolation*, 1930-1937.



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[1] Although we do not include vault cash under the definition of money, we must nevertheless take account of it because it enters as a negative factor into the supply of deposits; *cf.* section (4.4).

[2] J. W. Angell, *The Behaviour of Money*, New York and London, 1936.

It has been assumed that the same relation holds good for the demand for currency for payments in the following years, and that the magnitude of the idle hoards may therefore be estimated as the residual between the actual and the calculated M′. To obtain further evidence, the calculation has been continued through 1937 (Hoarding, estimate 1). It is possible that the trend movement in favour of the cheque has not continued at the same rate after 1929 as before that year. A probably extreme alternative has therefore been calculated, where the trend term in (4.611) was supposed to be nil for the period after 1929 (Hoarding, estimate 2).

A third estimate was made according to a principle indicated by Bertrand Fox.[1] Mr. Fox assumes that hoarding started in November 1930 and that it was not effected in $1 notes or coin. So the amount of hoarding may be determined by comparing the variations in the value of outstanding notes of denominations of $5 and over, with those of the $1 notes (Hoarding, estimate 3).[2] The result of the three estimates is shown below and in graph 4.612.[3]

Hoarding	1930	1931	1932	1933	1934	1935	1936	1937
	Milliards of dollars							
Estimate 1 . . .	0.09	0.89	2.08	2.50	1.96	2.02	2.28	2.53
Estimate 2 . . .	0.02	0.74	1.85	2.20	1.58	1.57	1.76	1.93
Estimate 3 . . .	0.03	0.60	1.36	1.50	1.05	1.00	1.13	1.16

---

[1] "Seasonal Variation in Money in Circulation", *Review of Economic Statistics* XXI, February 1939, pages 21-29.

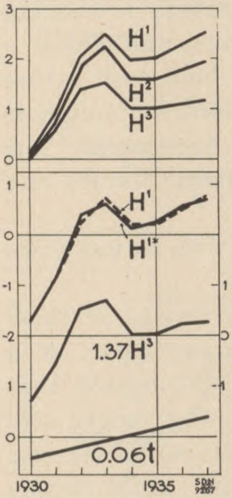
[2] The figures obtained by this procedure differ largely from those given by Mr. Fox, owing to the fact that we do not follow his assumption that hoards had been liquidated by January 1935. The argument offered in favour of this assumption — viz., that after this date "the movements of all denominations conform to the same pattern, and in turn, roughly to that of general business" (page 27) — only proves, it would seem, that there was no more *new* hoarding after that date, but not that the existing hoards had been liquidated.

[3] In an earlier publication of the League of Nations, *Commercial Banks 1925-1933* (Geneva, 1934), the amount of hoarding was estimated according to virtually the same method. The result, that "the actual amount of hoarding in June 1932 was . . . at least $1,600 million and probably more" (page 247), agrees with the present estimates.

It will be seen that the third estimate shows a good correlation with the first and the second, but with a smaller amplitude and a slight trend difference. This would suggest that there was still some hoarding of \$1 notes. On the basis of the correlation between estimate 1 and estimate 3, shown in graph 4.612, lower part, the hoards of \$1 notes may be estimated at about 3% of total hoarding.[1] This would not seem to be unreasonably large, and we may take our estimate 1 as final.

Graph 4.612.

Three Estimates of CURRENCY HOARDING, 1930-1937, and "Explanation" of the Fluctuations of H^1 by those of H^3 and *t*.



As we shall see in section (4.9), hoarding has a considerable influence on *n*. It is therefore of importance for the system of equations to include an "explanation" of hoarding.

This, however, raises a number of theoretical and statistical difficulties.

(i) Though there is a certain systematic, cyclical background to the phenomenon of currency hoarding, this variable, perhaps more than any other in our system, will be influenced by incidental factors. Hence we may expect large residuals in any "explanation" that is based only on endogenous factors.

(ii) The number of observations that may serve for the "explanation" is small. Hoarding started in the fourth quarter of 1930. The "explanation" by endogenous factors cannot go, it would seem, beyond the middle of 1933, when, as a consequence of the measures of bank control following the general bank holiday in March of that year, and

---

[1] In October 1930, the value of \$1 notes outstanding was about one-tenth of the value of all notes. Hence $0.1 \times 0.37\ H^3$, or $0.1 \times 0.37 \times \frac{100}{137} H^1$, or about 3% of H^1, kept in the form of \$1 notes, would be sufficient to explain the difference between the two estimates.

the somewhat improved business situation, the fear of bank failures might have ceased.[1]

The more or less stable amount of hoarding after that date should probably be ascribed to the fact that, possibly under the influence of the New Deal, a new equilibrium situation had developed, where hoards of some 2 milliard dollars were considered as normal. Our explanation must thus be restricted to the last quarter of 1930, the years 1931 and 1932, and the first two quarters of 1933. These five observations [2] clearly do not allow of a choice between different possible explanatory variables on the basis of a correlation calculus.

(iii) Different factors may have co-operated in causing the increasingly difficult position of the banks, and hence a rising distrust and an increasing tendency to hold cash rather than deposits. Apart from withdrawals of deposits by foreigners and hoarding itself, the following factors are mentioned: "the fall in commodity prices, security and real-estate values and personal incomes ".[3]

(iv) Each of these factors may have acted with an unknown but certainly not very large lag.

(v) It is not quite clear whether we must choose, of these explanatory variables, the actual value in any year, or a sum over some preceding period. It may be argued that the position of the banks becomes weaker, the longer bad trade continues — this would be a point in favour of the use of a sum; — or, on the contrary, that at any unfavourable

---

[1] Bank failures, which had involved a yearly loss of deposits of about $100 million to $300 million from 1921 to 1929, reached their peak in 1933 with a figure of $3,600 million of deposits involved. After that year, they were reduced to a negligible amount. Evidently bank failures are closely connected with hoarding, both as a cause and as an effect. But since the explanation of this phenomenon meets with the same difficulties as that of hoarding, it cannot give much help in the explanation of the latter.

[2] Nothing is gained by using eight quarterly figures for 1931 and 1932, because for almost any explanatory variable these eight values lie practically on a straight line. The heavy fluctuations in hoarding from one quarter to another are admittedly not due to the endogenous explanatory factors to be used.

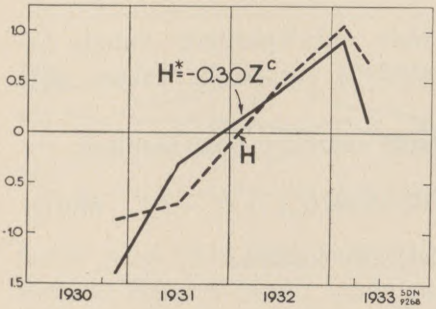
[3] *Commercial Banks 1925-1933*, League of Nations, page 246.

cyclical situation, a certain number of banks fail, but that the others will be able to continue however long this situation may last. Again, hoarding may correspond to the current rate of bank suspensions, or to the cumulated total of such failures over a certain time.

Lacking both theoretical and statistical evidence, the choice may be determined by considerations of a practical nature. Only one explanatory variable will be used — viz., Z^c, corporation profits, which is a good indicator of the business situation and, at the same time, most easy to handle in the elimination process (Chapter VI). A possible small lag is neglected. On practical grounds, too, Z^c rather than $\int Z^c$ (accumulation) is chosen.[1]

Graph 4.613.

“Explanation” of Fluctuations in CURRENCY HOARDING.



It appears from the data before 1930 that relatively small fluctuations in profits do not lead to hoarding or dishoarding. Also, the evidence since 1934 does not suggest, after a period of hoarding, a clear tendency to diminish hoards when business improves. From these facts it would follow[2] (i) that a low value of Z^c entails hoarding, but a high value no dishoarding; (ii) that the depression

---

[1] We may be pretty certain that the choice of any other possible explanatory variable — *e.g.*, (U + V) — would have given an only slightly different result.

Trials have also been made with the short-term rate of interest as the explanatory factor. The results were less convincing from a statistical point of view than those obtained with Z^c; and since there is no reason to believe that, in this particular country and period, the low rate of interest was the most important factor making for hoarding, the “explanation” by Z^c has been given preference.

[2] It must be admitted that the evidence from these last years, in which external factors may have played a large part, cannot be regarded as very conclusive.

must be rather serious — *i.e.*, that Z^c must have fallen a considerable amount below the preceding boom value, before hoarding starts. If we estimate this threshold value by comparing the profits in 1929 with those in the third quarter of 1930 — when hoarding had not yet appeared — we find that the minimum fall must be about 7 milliard dollars. Indicating by Z_m^c the maximum of the preceding boom, hoarding would occur when

$$Z_m^c - Z^c - 7 > 0.$$

The explanation of hoarding with Z^c over the period indicated yielded:

$$H = -0.30Z^c \quad (\textit{cf.}\text{ graph } 4.613).$$

This formula may be generalised so as to cover also years with increased or slightly decreased profits, by writing

$$H = 0.30\,(Z_m^c - Z^c - 7)'', \tag{4.612}$$

where the sign $''$ indicates that only positive values of the expression between the brackets are to be taken into account.

The general formula for outside currency now becomes:

$$M' = 0.043\,(L_w + L_s + E'_F) - 0.076t + 0.30\,(Z_m^c - Z^c - 7)'' \quad (4.61).$$

(4.62) Vault cash (VC) is statistically known:

(i) by weeks for reporting member banks in 101 cities;

(ii) on three or four call dates for all member banks; and

(iii) on June call dates for all banks.

As the function of vault cash is that of a small buffer stock, which is liable to relatively heavy fluctuations, not too much evidence can be gained from one, or even four, figures in a year.

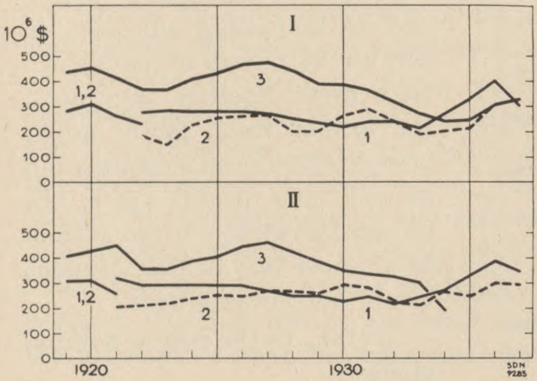
Graph 4.62 shows that the figures (or estimates) [1] that may be given for the three groups, reporting member banks, other member banks and non-member banks, for June call dates (or thereabouts) and for yearly averages are not very parallel. Moreover, the graph does not suggest, as might have been expected, a correlation with deposits. This lack both of reliable data and of a pronounced movement in what material is available, suggests that the variations in VC should be disregarded, and this item should be considered as a constant. This may be done the more readily since the variation of the figures is not large in absolute terms. It is possible that cash in vault had a tendency to be somewhat larger in the years 1934-1937, when the banks' excess reserves with the Federal Reserve Banks made such an increase cost very little. But the amount withheld from the reserves for this purpose was too small to have any influence on the short-term rate of interest (*cf.* section (4.42)).

Graph 4.62.

" Cash in Vault " of

1. Reporting member banks.
2. Other member banks.
3. Non-member banks.

I. At June call dates.
II. Yearly average.



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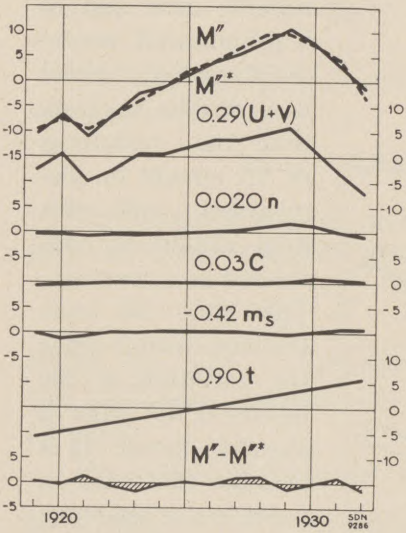
[1] *June call dates.* All banks and member banks given in the report of the Controller of the Currency; the last week in June is taken for the reporting member banks.

Yearly averages. Reporting member banks: average of 12 monthly figures. Member banks: average of three or four call-date figures. Non-member banks: total for all banks (derived from Angell, *op. cit.*, page 178: Outside currency, and from the Federal Reserve Banks' balance-sheets: Currency in circulation) minus the figure for member banks. The figures for non-member banks are, by this procedure, also slightly influenced by the difference between Angell's and our way of estimating the yearly figures for member banks.

(4.63) M″. The following factors have been included in the "explanation" of the demand for holding deposits.

Graph 4.63.

"Explanation" of the Fluctuations in Total Deposits by Demand Factors.



(i) (U + V), as an indication of the value of general business activity.

(ii) C′, the total market value of all shares, as an indication of the need for means of payment for speculative purposes.

(iii) t, indicating the net result of the increasing possibility to hold idle money as a consequence of increasing wealth and an increasing efficiency in the use of means of payment.[1] In a more rigorous treatment, the former trend factor might be considered as an accumulation of, say, past profits ($\int Z^c$); but for our knowledge of the system as a whole this further element would not be important.

(iv) m_S, as the cost of holding money.

These factors yield the following regression equation:

$$M'' = 0.29\,(U + V) + 0.018C' - 0.42m_S + 0.90t,$$

or, after substitution of an expression in C and n for C′:

$$M'' = 0.29\,(U + V) + 0.03C + 0.020n - 0.42m_S + 0.90t \qquad (4.63).$$

---

[1] A positive secular trend is much more characteristic of the older data available for deposits than the cyclical movements. The tremendous secular increase in deposits is shown below (Angell, *op. cit.*, page 175):

1890	4.0	milliard	dollars
1900	7.1	,,	,,
1910	14.7	,,	,,
1920	36.7	,,	,,
1930	52.3	,,	,,

The elasticity of demand for deposits with regard to the rate of interest turns out to be very low:

$$0.42 \times \frac{\bar{m}_S}{M''} = 0.045.$$

This is comprehensible owing to the fact that the amount of money necessary to effect payments is very inelastic in the short run.

(4.7) Demand by Individuals for Bonds and Shares

The relations determining the demand for holding bonds and shares by individuals are of the same structure as those determining the holding of assets by the banks (*cf.* section (4.5)).

The attractiveness of shares consists of two factors: the expected income (d, the rate of dividend for all companies) and the expectation as to the future course of share prices ($\dot{n}$, the rate of increase in share prices). The attractiveness of bonds consists in c (nominal rate of interest), which we have considered as a constant, and $\dot{m}_{Lb}$, which we shall disregard since we have found this factor to have a negligible influence on the banks' demand. Neither will m_S be taken into account; this implies the assumption that the influence of short-term credit conditions on the stock market is only very secondary. This has been stated by various authors — *e.g.*, Owens and Hardy [1] and Donner [2] — although it must be recognised that some authors — *e.g.*, Carl Snyder [3] — seem to be of a different opinion. The latter, however, only speaks of upper turning-points, and these will be treated in a special manner in our analysis.

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[1] Owens and Hardy, *Interest Rates and Stock Speculation*, Washington, 1930.

[2] O. Donner, "Die Kursbildung am Aktienmarkt", *Vierteljahreshefte zur Konjunkturforschung*, Sonderheft 36, Berlin, 1934.

[3] Carl Snyder, "The Problem of Monetary and Economic Stability", *Quarterly Journal of Economics*, XLIX, February 1935, page 200: "Speculation is acutely sensitive to these high rates of interest (having never survived the equivalent of a 6 per cent discount rate) . . ."

With these considerations, the demand for shares and bonds would depend on:

(i) A, the total wealth available for holding shares and bonds, which equals

$$\frac{c}{\bar{\bar{m}}_{Lb}} \bar{\bar{B}}^i + 0.0156\bar{\bar{n}}\,\bar{\bar{C}}^i;$$[1]

(ii) m_{Lb} and n, representing the prices of bonds and shares;

(iii) d and $\dot{n}$, representing the attractiveness of holding shares.

Thus we have the following demand equations:

$$C^i = \Gamma_1 A - \Gamma_2 m_{Lb} - \Gamma_3 n + \Gamma_4 d + \Gamma_5 \dot{n} \qquad (4.71)$$

$$B^i = B_1 A + B_2 m_{Lb} + B_3 n - B_4 d - B_5 \dot{n}, \qquad (4.72)$$

where the unknown coefficients are represented by the letters Γ_1 . . . and B_1 . . . . These letters represent positive figures everywhere.

To these two equations the definition equation of A may be added, which, in deviations from average, runs as follows:[2]

$$A = 0.90B^i + 1.50C^i - 18.0m_{Lb} + 0.84n \qquad (4.73).$$

It may be useful, at this stage, to give some thought to (*a*) the purpose of the system of monetary equations[3] and (*b*) the way in which this purpose may be attained.

(*a*) The equations discussed in this chapter describe the financial sphere of the economy. The variables explained in these equations enter into the other relations of the system in a few places only: the share price index n in the explanation

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[1] The factor 0.0156 must be included, since the absolute price level of shares in terms of nominal value was 1.56 in 1926, when the share price index stood at 100.

[2] *Cf.* section (4.5). The averages of B^i, C^i, m_{Lb} and n are 100, 54, 5 and 96 respectively.

[3] *I.e.*, the equations with regard to demand and supply in the money and capital markets.

of G (5.3), the share yield $m_{\mathrm{L}s}$ $(=\frac{d}{n})$ in the explanation of v' (2.4) and the bond yield $m_{\mathrm{L}b}$ in the explanation of v_{B} (2.5); the influence of the short-term rate of interest m_{S} on stocks was not found to be appreciable (2.6). What is needed, therefore, in order to find the movements of the essential variables of our system, is the expression of n and $m_{\mathrm{L}b}$ in variables not belonging to the monetary sphere. This is, in the frame of our analysis, the *raison d'être* of the present chapter.

(*b*) To find these expressions for n and $m_{\mathrm{L}b}$, the system of monetary equations must be determined — *i.e.*, must contain as many independent equations as monetary variables. To what extent, and how, is this the case ?

If we disregard, for a moment, the banks' demand for holding bonds, we shall have:

2 supply equations for B^i and C^i;
2 demand equations for B^i and C^i;
1 definition equation for A.

As the right-hand members of the two demand equations must, after multiplication by the corresponding prices, add up to the expression for A, one of the three last-mentioned equations is dependent on the two others; so we have all together four independent equations. With these, we have to determine the five unknowns A, B^i, C^i, n, $m_{\mathrm{L}b}$: our problem is undetermined. The best thing we can find with our four equations is *one* equation connecting *two* unknowns and containing further some non-monetary variables. For instance, we may find an equation expressing n in $m_{\mathrm{L}b}$ and d, $\dot{n}$, etc.

On second thoughts, there is nothing peculiar in this result. If the subjects of one group hold bonds and shares, and nothing but bonds and shares, they will show a certain readiness to exchange one asset for the other (or, in other words, an indifference curve for the two assets), depending on their relative attractiveness; but there will not be a price in terms of money, since the group is supposed not to change the assets for money.

This isolation of bonds and shares from money and all other goods that are measured in terms of money will be broken if at least one of the two assets is held by at least one group of subjects that also holds money. Under our simplifications, it is the banks which establish this link.

Hence our plan becomes as follows:

(i) Determine the coefficients in one of the two interdependent equations (4.71) and (4.72);

(ii) In equation (4.71), which is then known with numerical coefficients, substitute C^i as given by its supply equation (*cf.* section (4.2));

(iii) Substitute in (4.73) the expressions given by the supply equations for bonds and shares and the definition equation:

$$B = B^i + B^b.$$

This yields, if we take the supply equations in the simplified form of trends:[1]

$$A = 8.0t - 0.9B^b - 18.0m_{Lb} + 0.84n \qquad (4.74).$$

(iv) Eliminate A from (4.71) and (4.74).

In this way we obtain a relation (4.75) between n, m_{Lb} and B^b as monetary variables on the one hand, and the non-monetary variables d, $\dot{n}$ and t on the other hand. The combination of this equation with the four demand equations for M'', M', B^b and B_s and the two supply equations of M and B_s, gives us the seven equations to determine the seven monetary variables they contain: M'', M', B^b, B_s, m_S, m_{Lb}, n.

Unfortunately, in the execution of this plan, we are repeatedly faced with serious multicollinearities which prevent us from determining some of the coefficients with any degree of precision. A way to overcome this difficulty consists in reducing,

---

[1] *Cf.* sections (4.1) and (4.2).

$$C^i = 0.755C = 0.755 \times 3.18t = 2.40t.$$

by the combination of a number of theoretical equations, the number of highly intercorrelated variables to only one, and performing the correlation calculation for the equation so found (which is the result of an elimination process) instead of for the elementary equations. We shall have recourse to this procedure every time the difficulty of multicollinearity presents itself.

Equation (4.71) is the first example. During the period studied, the movements of n were so large that they almost entirely determined the fluctuations in A. Moreover, n and a combination of d and $\dot{n}$ are very highly intercorrelated. Hence we jump (i), execute (ii), (iii) and (iv) and find (4.75) with coefficients that still contain the Γ_s' from (4.71):

$$-(\Gamma_3 - 0.84\,\Gamma_1)n - (18.0\,\Gamma_1 + \Gamma_2)m_{\mathrm{L}b} - 0.9\,\Gamma_1 \mathrm{B}^b + \Gamma_4 d + \Gamma_5 \dot{n} + (8.0\,\Gamma_1 - 2.40)\,t = 0 \qquad (4.75).$$

The six coefficients in this equation should now be found by a correlation calculation. But since in this correlation the terms with n, d and $\dot{n}$ are most important and the rôle of B^b and the trend is subordinate, and since, moreover, B^b is highly correlated with t, the determination of the coefficient for B^b is rendered illusory. Hence we provisionally[1] disregard the deviations that B^b shows from a trend, and use the purely statistical approximation:

$$\mathrm{B}^b = 0.63t \quad (\mathrm{R} = 0.918).$$

Further, to render the interpretation easier, we write the equation obtained by this substitution, with n explicit:

$$n = \frac{1}{\Gamma_3 - 0.84\,\Gamma_1}\,[\Gamma_4 d + \Gamma_5 \dot{n} - (18.0\,\Gamma_1 - \Gamma_2)\,m_{\mathrm{L}b} + (7.4\,\Gamma_1 - 2.40)t] \qquad (4.76)$$

as the "explanation" of the share price.

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[1] *Cf.* page 112 note 1.

(4.8) THE SHARE-PRICE EQUATION

The equation for n derived in the previous section may, with simplified coefficients, be written as follows:

$$n = \nu_1 d + \nu_2 m_{Lb} + \nu_3 \dot{n} + \nu_4 t \qquad (4.81).$$

In this equation, the exact dependence of n on $\dot{n}$ has in particular been studied. It became evident that the fit could be improved by introducing a non-linear and even a third-degree dependence on $\dot{n}$, together with a small lag of about half a year. [1] This function — which at the same time represents the functional dependence between the demand for holding shares and $\dot{n}$ — is represented graphically in graph 4.81. It seems to show that, as long as $\dot{n}$ is not extreme, no large influence on holdings is present; but this influence becomes increasingly large, especially for positive values of $\dot{n}$. This evidently indicates what one might call "the speculative attitude" or "the boom psychology".

The numerical expression for n is:

$$n = 19.5d - 9.1m_{Lb} + 0.025\,(n - n_{-1})^2 + 0.00035(n - n_{-1})^3 + 0.55t \qquad (4.82).$$

The linear approximation is:

$$n = 26.9d + 6.8m_{Lb} + 0.26\,(n - n_{-1}) + 3.88t \qquad (4.83).$$

The approximation without $\dot{n}$ is:

$$n = 29.9d + 4.0m_{Lb} + 5.7t \qquad (4.84).$$

It will be seen that the coefficient for m_{Lb} gets the wrong sign in (4.83) and (4.84); but this sign is not significant, as the

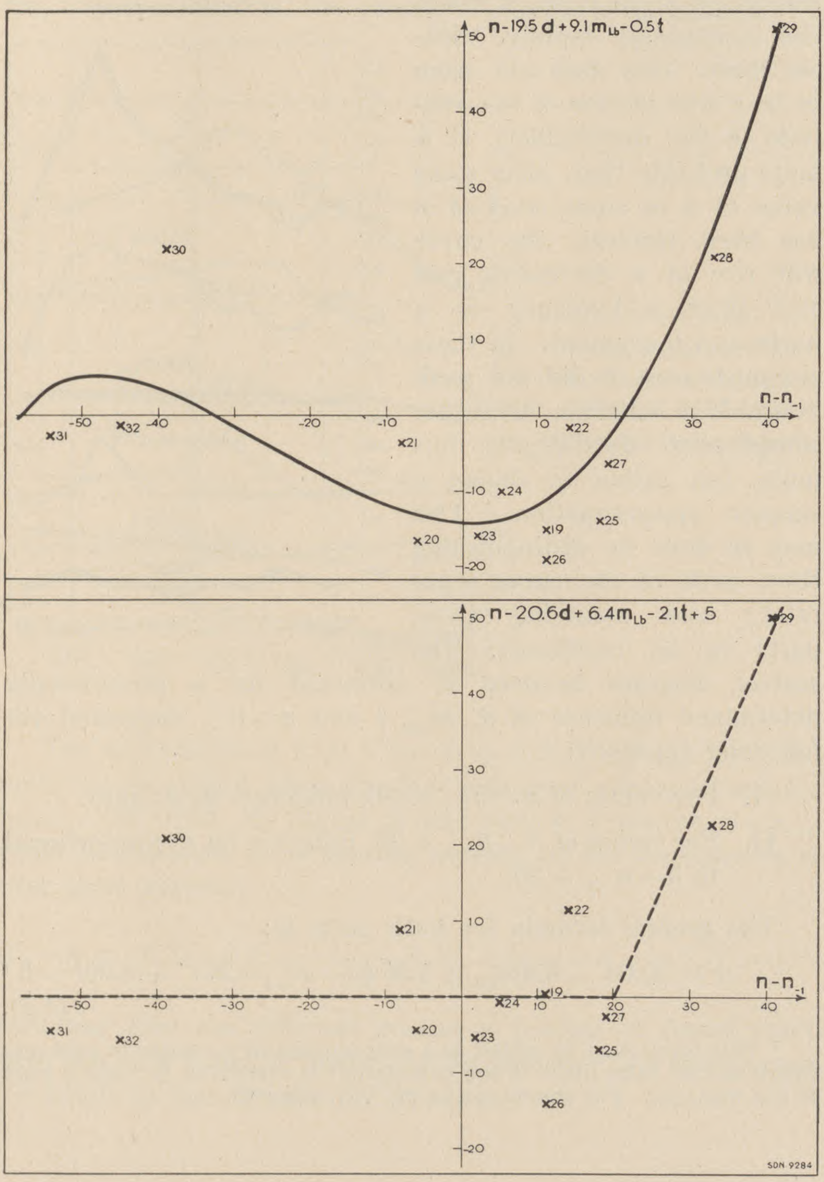
---

[1] As a first approximation, this lagged value of $\dot{n}$ may be taken equal to $n - n_{-1}$. A still better approximation is of course $n_{-\frac{1}{3}} - n_{-\frac{2}{3}}$, especially as this does not contain n_0 itself, which has to be "explained". See below.

Graph 4.81.

PARTIAL SCATTER DIAGRAM BETWEEN n AND $\dot{n}_{-\frac{1}{2}}$.

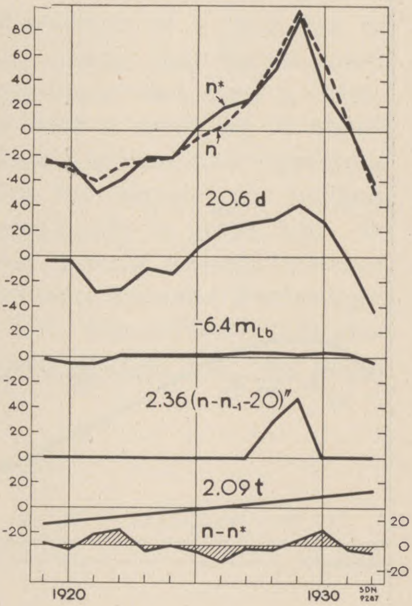
× × × Actual values, corrected for influence of d, m_{Lb}, and t;
——— Third-degree curve;
- - - - - Approximation by linear parts.



standard error of the coefficient is much larger than the coefficient itself: 12.6 in (4.83) and 12.7 in (4.84).

Graph 4.82.

"Explanation" of Fluctuations in SHARE PRICES.



There is, however, a theoretical objection against the third-degree function used. If extrapolated to the right, it would rise increasingly rapidly, without limit. This does not seem to be a true picture of the attitude of the shareholder. It is more probable that, after some value of $\dot{n}$ or some level of n has been reached, the curve will rise at a decreasing rate and show a tendency to a horizontal movement. In these circumstances, it did not seem desirable to maintain the rather complicated third-degree formula, but rather to choose a simpler approximation. This may be done by distinguishing three parts of the curve separately and assuming these parts to be rectilinear. The scatter diagram between n, corrected for a provisionally determined influence of d, m_{Lb}, t and $n-n_{-1}$ suggested the following approach:

I. For values of $n-n_{-1} < 20$, influence on n: zero.

II. For values of $n-n_{-1} > 20$, influence on n: proportional to $n-n_{-1}-20$.

The general formula for both parts is:

$n = 20.6d - 6.4m_{Lb} + 2.36\,(n-n_{-1}-20)'' + 2.09t - 5$[1]

(4.8).

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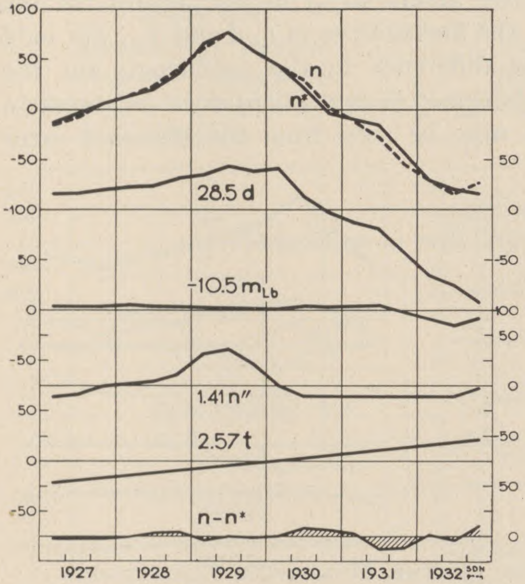
[1] This term must be added as a consequence of a change in averages which has to take place if the calculation is restricted to only a part of the material. For the meaning of ″, *cf.* page 98.

The scatter diagram did not show very clearly on what level the third, horizontal branch should be chosen. As the maximum monthly value for n (corrected for d, m_{Lb} and t) equals about 100, it must have been at about that level or higher. This approximation of the curve by linear parts offers some convenience for the treatment of the problems considered in Chapter VI.



Graph 4.83.

"Explanation" of Fluctuations in SHARE PRICES, 1927-1932 (Four-monthly periods).

The dependence of n on $\dot{n}$ has been tested with shorter time units for the period where the fluctuations of n are particularly heavy: 1927-1932. By the use of time units of 4 months, $n - n_{-1}$ could be replaced by a moving average of the increases in n over these periods:

$$(n_0 - n_{-\frac{1}{3}} - 6.7)'' + (n_{-\frac{1}{3}} - n_{-\frac{2}{3}} - 6.7)'' + (n_{-\frac{2}{3}} - n_{-1} - 6.7)'' = n'',$$

assuming that the different groups of holders of shares react with lags of 2, 6 and 10 months to the increases in n that exceed 6.7 points in four months (20 points a year).

The result of an explanation with d, m_{Lb}, n'' and t over the eighteen time units considered runs as follows:

$$n = 28.5d - 10.5m_{Lb} + 1.41n'' + 2.57t \qquad (4.85).$$

The most important difference from (4.8) is the decrease in the coefficient for n'' as compared with that for $(n - n_{-1} - 20)''$. It is largely due to the fact that the fluctuations of n'' are considerably accentuated (by about 40%) in the figures for the shorter period, whereas the fluctuations of n, d and m_{Lb} are only slightly increased. The difference in the coefficients for the latter two variables between (4.8) and (4.85) does not seem to be very significant, as may be seen from the standard error of the coefficients:

Standard error of coefficients

Equation	σ_d	$\sigma_{m_{Lb}}$	$\sigma_{(n-n_{-1}-20)''}$, $\sigma_{n''}$
(4.8)	4.3	8.8	0.63
(4.85)	2.9	5.7	0.22

(4.9) COMBINATION OF THE MONETARY EQUATIONS

Equation (4.8) gives one relation between the two monetary variables n and m_{Lb} and some non-monetary variables d, $(n - n_{-1} - 20)''$, t. Combining the other monetary equations, as planned in section (4.7), we find another equation between n and m_{Lb}, which contains in addition the variables (U + V), d, Au, P, $(L_w + L_s + E'_F)$, H, t.[1] (The details of this elimination process are shown in Appendix B). These two equations serve to express both n and m_{Lb} in the non-monetary variables:

---

[1] Au and P, though monetary variables, may remain in this elimination result because they are considered as data.

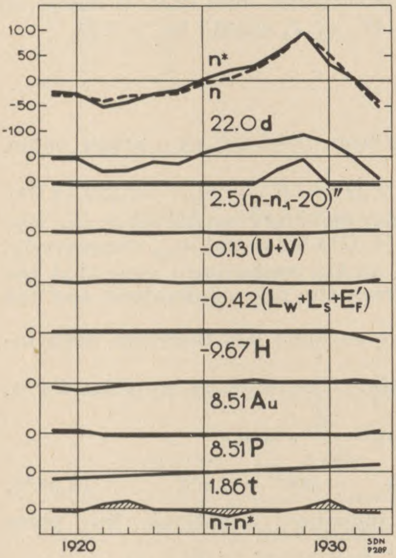
H (hoarding) has not yet been reduced to its explaining variable (4.612), in order to simplify the formula; it is better to do this later.

$$\begin{aligned} n = {} & 22.0d + 2.5(n - n_{-1} - 20)'' - 0.13(\mathrm{U} + \mathrm{V}) \\ & - 0.42(\mathrm{L}_w + \mathrm{L}_s + \mathrm{E}'_\mathrm{F}) - 9.67\mathrm{H} + 8.51(\mathrm{A}u + \mathrm{P}) \\ & + 1.86t \end{aligned} \tag{4.91}$$

$$\begin{aligned} m_{\mathrm{L}b} = {} & - 0.22d - 0.02(n - n_{-1} - 20)'' + 0.02(\mathrm{U} + \mathrm{V}) \\ & + 0.065(\mathrm{L}_w + \mathrm{L}_s + \mathrm{E}'_\mathrm{F}) + 1.51\mathrm{H} - 1.33(\mathrm{A}u + \mathrm{P}) \\ & + 0.04t \end{aligned} \tag{4.92}$$

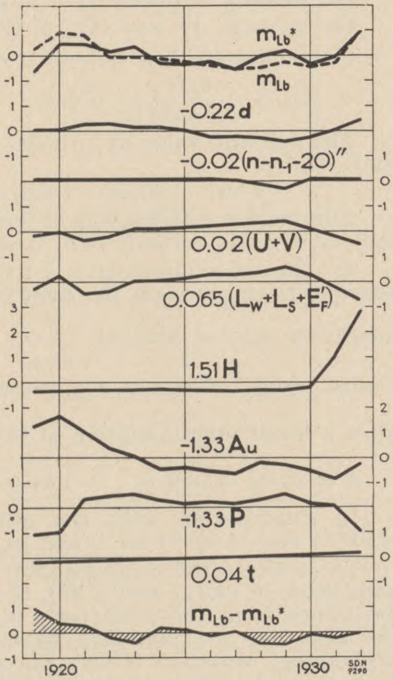
Graph 4.91.

ELIMINATION RESULT (Monetary Equations): *n* expressed in Non-monetary Variables.



Graph 4.92.

ELIMINATION RESULT (Monetary Equations): $m_{\mathrm{L}b}$ expressed in Non-monetary Variables.



Graphs 4.91 and 4.92 show the fit of the two formulæ. For n the result is satisfactory. For $m_{\mathrm{L}b}$ it is much less good,[1]

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[1] Owing to the fact that here the residuals in the elementary relations have been multiplied by relatively large coefficients in the course of the elimination process.

but since m_{Lb} only enters, and with a small coefficient, into the v_B-equation, some uncertainty with regard to the m_{Lb}-equation is not a serious matter for the system.[1]

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[1] Two alternatives may be considered.

1. $0.9\,\Gamma_1 B^b$ has been replaced by a trend (*cf.* page 105). We may now determine the possible consequences of this simplification. The numerical value of the coefficient that B^b ought to have in (4.76): $\dfrac{-0.9\,\Gamma_1}{\Gamma_3 - 0.84\,\Gamma_1}$ may be known within certain limits, given:

(i) the coefficient found for t in (4.8), which yields:

$$\frac{7.4\,\Gamma_1 - 2.4}{\Gamma_3 - 0.84\,\Gamma_1} = 2.09; \text{ and}$$

(ii) $1.00 > \Gamma_1 > 0.35$. An increase in wealth will be distributed partly in shares, partly in bonds; hence Γ_1 cannot be > 1. When the total wealth becomes greater, there will be a tendency to increase the holding of shares by a larger percentage than that of bonds; in the average, $\overline{C}^i$ was 35% of $\overline{C}^i + \overline{B}^i$, so Γ_1 should be > 0.35.

From (i) and (ii) it follows that

$$3.5 > \frac{0.9\,\Gamma_1}{\Gamma_3 - 0.84\,\Gamma_1} > 0.37.$$

The extreme value of $-\ 3.5B^b$ has been tried out as *a priori* value, and yielded

$$n + 3.5B^b = 20.4d - 1.9m_{Lb} + 2.37\,(n - n_{-1} - 20)'' + 4.52t \quad (4.9^*).$$

Elimination with the help of the other monetary equations in the way indicated yields variants (4.91*) and (4.92*) for n and m_{Lb} respectively.

2. The other alternative is based on the well-known view that the share price depends on the *ratio* between the rate of dividend and the long-term rate of interest: $\dfrac{\overline{\overline{d}}}{\overline{\overline{m}}_{Lb}}$. This variable has therefore been included instead of d and m_{Lb} separately. The resulting equation for n, after a linear approximation of the ratio $\dfrac{\overline{\overline{d}}}{\overline{\overline{m}}_{Lb}}$, runs:

$$n = 16.0d - 18.9\,m_{Lb} + 2.64\,(n - n_{-1} - 20)'' + 1.53t \qquad (4.9^{**}).$$

In combination with the other monetary equations, this gives (4.91**) and (4.92**) for n and m_{Lb}.

To facilitate the comparison of the coefficients of these alternatives with those of (4.91) and (4.92), the former are expressed as percentage deviations from the coefficients in these latter cases. Since, however, the influence (= standard deviation × coefficient) of the various explanatory variables is very unequal — as the graphs show — a like percentage deviation is not equally important for all variables. As an indication of these differences, the influence of each explaining variable in (4.91) and (4.92) is added, expressed as a percentage of the standard deviation of the "explained" series (table 4.9).

The results are satisfactory in that the series with the largest influence, both for n (d; $(n - n_{-1} - 20)''$) and for m_{Lb} (Au, P; $L_w + L_s + E'_F$, H), show rather stable coefficients. Again, the results are less good for m_{Lb} than for n.

Note continued on page 113.

From the m_{Lb} - equation, the effects of open-market policy on the long-term rate of interest may be determined. The factor —1.33, which multiplies P, indicates that a \$1 milliard increase of the Federal Reserve Banks' holdings of bonds or acceptances leads, *ceteris paribus*, to a fall in the long-term rate of interest of 1.33%. The two alternatives treated give coefficients for P quite near to this value: —1.37 and —1.48.

Table 4.9.

"Explained" variable	Equation	Unit	Coefficients for:					
			d	$(n - n_{-1} - 20)''$	U + V	Au+P	$(L_w+L_s+E'_F)+H$	t
n	(4.91*)	Percentage deviations from coefficients in (4.91)	+ 10	+ 9	+ 328	—125	— 69	— 6
	(4.91**)		— 10	+ 26	+ 257	+ 236	+ 234	— 83
	Influence of variables, in % of standard deviation of n							
	(4.91)	$\sigma_n = 100$	63	36	— 4	20	— 19	18
m_{Lb}	(4.92*)	Percentage deviations from coefficients in (4.92)	—632	—784	—140	+ 3	— 14	+ 725
	(4.92**)		— 8	+ 26	+ 15	+ 11	+ 11	0
	Influence of variables, in % of standard deviation of m_{Lb}							
	(4.92)	$\sigma_{m_{Lb}} = 100$	— 50	— 22	52	—258	242	32

These figures point to the conclusion that the Federal Reserve Banks are able, in view of the small year-to-year changes in m_{Lb},[1] to control to a large extent the fluctuations of the long-term rate of interest by means of not excessively large open-market purchases or sales (in a period when there are no large excess reserves).

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[1] Distribution of year-to-year changes in m_{Lb}, in percentages, 1920-1937:

0 *to* $\frac{1}{4}$	$\frac{1}{4}$ *to* $\frac{1}{2}$	$\frac{1}{2}$ *to* 1	*Over* 1
11	3	3	1

CHAPTER V

DESCRIPTION OF THE MODEL. INCOME FORMATION

(5.1) "Explanation" of Dividend Fluctuations

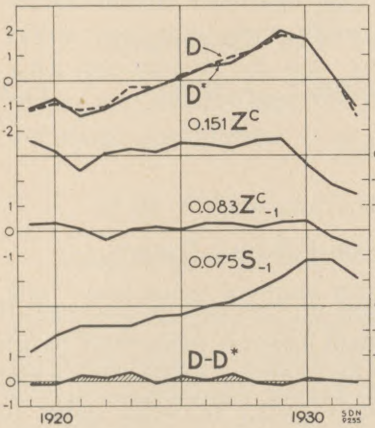
Dividends may be expected to be chiefly determined by profits and reserve position. Both factors may work with some lag. The relation to be tested has therefore been given the form:

$$D = \delta_0 Z^c + \delta_1 Z^c_{-1} + \delta_2 S_{-1}$$

It is not necessary to include S, as S will be dependent on S_{-1}, Z^c and D.

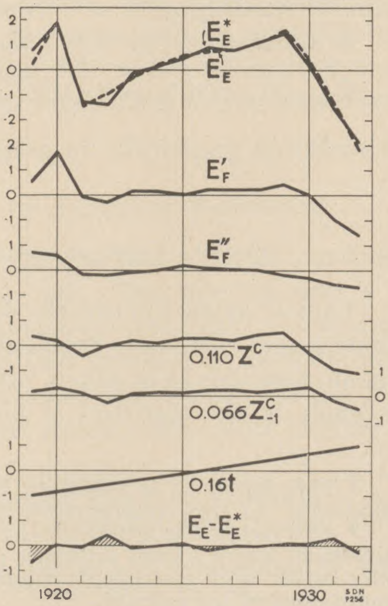
Graph 5.1.

"Explanation" of Fluctuations in Dividends.



Graph 5.2.

"Explanation" of Fluctuations in Entrepreneurial Withdrawals.



The result of the testing is given in equation (5.1) and graph 5.1. A high correlation is found, and a rather high influence of the reserve position.[1] Relation (5.1) runs:

$$D = 0.151Z^c + 0.083Z^c_{-1} + 0.075S_{-1} \qquad (5.1).$$

(5.2) "EXPLANATION" OF ENTREPRENEURIAL WITHDRAWALS

Entrepreneurial withdrawals are only roughly estimated. Very refined experiments with these figures do not, therefore, seem possible. First, farmers' incomes (in money and in kind) were subtracted. It seemed natural to assume as the chief influencing factors for the remaining incomes:

(i) The general profit situation, which may be best characterised by corporation profits Z^c, and

(ii) A trend, representing changes in reserves.[2]

The influence of the first variable might be lagged, as corporations are probably representative of the more exposed and rapidly reacting part of business life.

A satisfactory fit was obtained with the formula:

$$E_E - E'_F - E''_F = 0.110Z^c + 0.066Z^c_{-1} + 0.16t \qquad (5.2)$$

represented graphically in graph 5.2.

(5.3) "EXPLANATION" OF CAPITAL GAINS

Capital gains will chiefly depend on the rate of increase in share prices. The only problem which arises is over what period the increase has to be taken. Judging from the distinction which is made in the statistics of income — viz., between gains on

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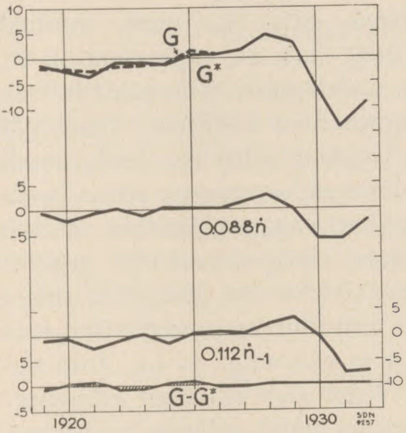
[1] This influence is found to be much smaller in some European countries. *Cf. De Nederlandsche Conjunctuur*, August 1935.

[2] This factor was introduced by analogy with the case of corporation dividends where a large influence of surplus was found. Surplus shows only rather slow movements which, over the period covered, may be approximated by a trend.



Graph 5.3.

" Explanation " of Fluctuations in CAPITAL GAINS.

(For the years 1930 to 1932 G has been taken equal to G* for lack of reliable figures.)

assets held less than two years and gains on assets held two years and more — considerable lags seem possible. Statistical investigation confirmed this view, and the best fit was obtained by the formula :

$$G = 0.088\,\dot{n} + 0.112\,\dot{n}_{-1} \qquad (5.3)$$

which means that the average period over which gains were taken was one year.[1] This is, of course, not in contradiction with the above, for the average will no doubt include both longer and shorter lags, the latter originating largely from stock-exchange speculation.

(5.4) "EXPLANATION" OF INTEREST PAYMENTS

Total interest payments are the product of "debt outstanding" and some average interest rate. This interest rate is an average of rates for various types of long-term debt[2] — *i. e.*, debts carrying various degrees of risk and incurred at various dates over a considerable period of previous time. Both factors tend considerably to smooth out fluctuations from year to year in

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[1] In fact, $0.088\,\dot{n} + 0.112\,\dot{n}_{-1}$ is very near to $0.20\,\dot{n}_{-0.56}$ (*cf.* page 46, note 1), which again is almost equal to $0.20\,\dot{n}_{-0.5} = 0.20\,(n - n_{-1})$. This expression would be obtained if all capital gains resulted from a holding of one year.

[2] Short-term interest payments have been considered as inter-business payments, as is done by Dr. KUZNETS, *loc. cit.*

this average interest rate. Hence only the most marked changes in business-cycle conditions find an expression in it, and even these are smoothed out and lagged. The same is true for the total of debts outstanding, where, in addition, a trend will be present. These two reasons, together with the fairly small size of the fluctuations in total interest payments, are a justification for applying only a rather rough procedure in the "explanation" of these movements. Only two rather general suppositions will be made, — viz.: (i) that the general business position, as measured by Z^c, exerts an influence, and (ii) that this influence is lagged and cumulative in character — *i.e.*, that the values of Z^c for many preceding years also exert an influence. The simplest mathematical expression which reflects both types of force is:

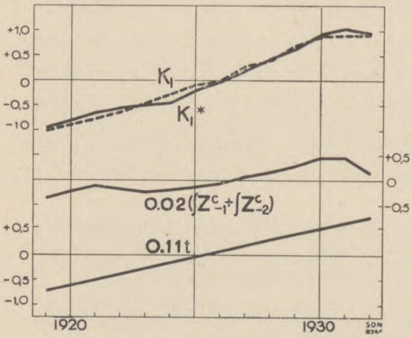
$$K_I = \varkappa_1 \int Z^c_{-1} + \varkappa_2 \int Z^c_{-2} + \varkappa_3 t$$

which has therefore been chosen for testing. The best fit has been found with

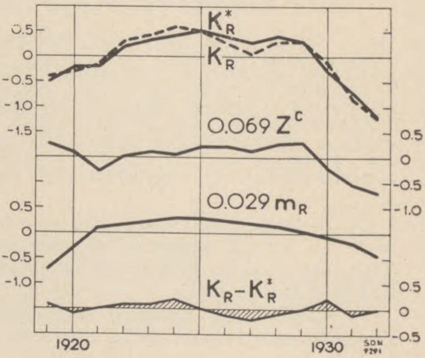
$$K_I = 0.020 \left(\int Z^c_{-1} + \int Z^c_{-2} \right) + 0.11t \qquad (5.4).$$

A trend has been added in order to account for secular changes, and for the purely mathematical reason that $\int Z^c$ is a sum of deviations, which differs from a simple sum by a trend term.

Graph 5.4.
"Explanation" of Fluctuations in Interest Payments.



Graph 5.5.
"Explanation" of Fluctuations in Rent Payments.



(5.5) "EXPLANATION" OF RENT PAYMENTS

Rent payments are also a minor income category, and are therefore considered only roughly. It would seem natural to assume two chief influences — viz., the general business position, most easily represented by Z^c, and the special position in the housing market, represented by m_R, rent level. The inclusion of these two factors gives a satisfactory approximation to this rather inexactly known income category. The relation found by correlation calculation is:

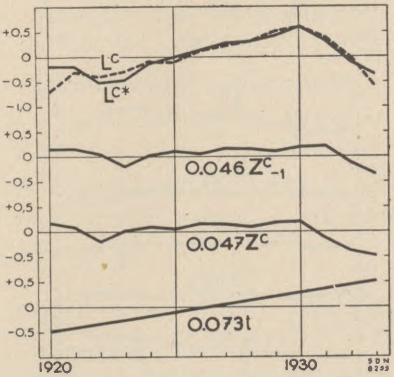
$$K_R = 0.069Z^c + 0.029m_R \qquad (5.5).$$

It is remarkable that no lag is found to exist in this relation.

(5.6) "EXPLANATION" OF CORPORATION MANAGERS' SALARIES

This category of incomes seems to depend directly on business profits, like dividends, probably with some lag. In addition, there is a structural tendency to growth in this group of incomes, which may be represented by a trend. A relation based on these assumptions was tried, and the best fit found was:

$$L_c = 0.047Z^c + 0.046Z^c_{-1} + 0.073t \qquad (5.6).$$



Graph 5.6.
"Explanation" of Fluctuations in CORPORATION MANAGERS' SALARIES.

(5.7) "EXPLANATION" OF LOWER SALARIES

The total amount of salaries could be regarded, as will be done in the case of wages, as the product of hourly salaries and the number of hours worked by all salary-earners. A further explanation ought then to be given of the number of hours and the hourly salaries. Salary-earners' employment, however, seems to be much less directly influenced by the volume of production than workers' employment; no doubt this is largely due to the "overhead" character of their work. The level of hourly salaries will depend chiefly on the profit situation and will be slow in its adaptation. Hence, instead of "explaining" employment and hourly earnings separately by about the same factors, it seemed preferable to explain the product of the two (for which, incidentally, better statistics are available) by profits with lags of 0,1 and (tentatively) 2 years, and a trend:

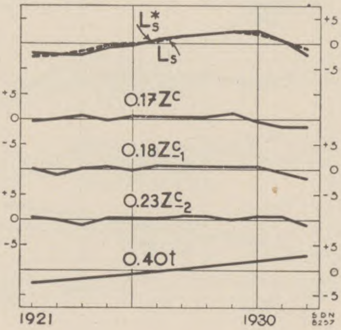
$$L_s = 0.170Z^c + 0.185Z^c_{-1} + 0.225Z^c_{-2} + 0.40t, \qquad (5.7)$$

$$L_s = 0.082Z^c + 0.368Z^c_{-1} + 0.37t \qquad (5.7').$$

The fit of (5.7) ($R = 0.990$) is somewhat, but not very much, better than that of (5.7′) ($R = 0.965$).

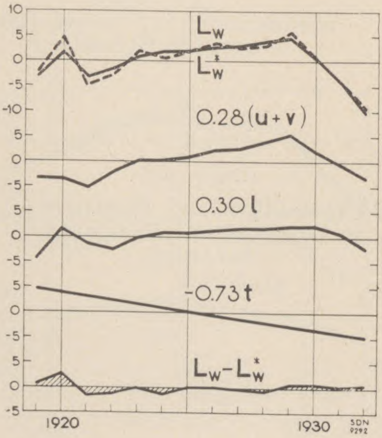
Graph 5.7

"Explanation" of Fluctuations in LOWER SALARIES.



Graph 5.8.

"Explanation" of Fluctuations in WAGES.



(5.8) "EXPLANATION" OF TOTAL WAGES

Total wages $(\bar{\mathrm{L}}_w+\mathrm{L}_w)$ are the product of the wage rate $(\bar{l}+l)$ by employment. Employment is closely connected with the volume of production as far as the shorter fluctuations are concerned; the long-run influence of changes in technique may be approximated by a trend. (We may disregard the dependence of this secular increase on the business cycle, which, partly because there are influences in the positive as well as in the negative direction, is only slight.)

The procedure followed consists in fitting an indirect estimate of employment $\frac{(\bar{\bar{\mathrm{L}}}_w)}{\bar{l}}$ with $(u+v)$ and a trend. The linear approximation of this result runs:[1]

$$\mathrm{L}_w = 0.28\,(u+v) + 0.30l - 0.73t \qquad (5.8).$$

(5.9) "EXPLANATION" OF DEPRECIATION ALLOWANCES

I. *Theoretical.*

Depreciation allowances will depend first on the value of capital goods in existence. This value is the sum of net additions during each year. Net additions will, in general, be large if gross additions are large. Gross additions being equal to V, and their sum represented by $\int \mathrm{V}$, this last variable must be included as one of the explanatory series.

If replacement were constant through time (say V_r), net investment would be equal to $\bar{\mathrm{V}} + \mathrm{V} - \bar{\mathrm{V}}_r$ and total capital, to the cumulation of this value; as the cumulation of a constant is a rectilinear trend series, total capital would be equal to $\int \mathrm{V}$ + a trend. Since the average duration of life may be taken at about 24 years,[2] depreciation allowances would have to be reckoned as $0.04 \int \mathrm{V}$ + a trend. If replacement moves parallel to V, the coefficient will be smaller than 0.04.

---

[1] The result is not changed appreciably if (as, strictly speaking, should be the case) $u+v$ is replaced by $u+v+u^e-u^i$, u^e and u^i representing the volume of exports and imports respectively.

[2] Calculated from data given by FABRICANT, Bulletin No. 60 of the National Bureau of Economic Research.

A second influence will be that of prices of capital goods q, especially with regard to repairs which are included in N (*cf.* graph 5.9). The influence of q would be much larger if entrepreneurs based their depreciation allowances on the principle of replacement cost — but this practice seems to be rare.[1]

A third influence will be that of the actual production [2] $u + v$. In good years, more will be charged than in bad years, when no allowances at all [3] may even be made. Thus, an equation of the following type is obtained:

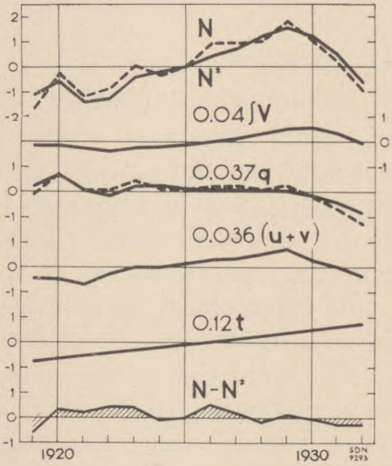
$$N = N_1 \int V + N_2 t + N_3 (u + v) + N_4 q.$$

II. *Statistical.*

A fairly good fit is found with the following equation:

$$N = 0.04 \int V + 0.12 t + 0.036 (u + v) + 0.037 q \qquad (5.9).$$

where the coefficient 0.04 for $\int V$ is taken *a priori*; the result of the correlation calculation was slightly, but not significantly, lower.



Graph 5.9.
"Explanation" of Fluctuations in DEPRECIATION ALLOWANCES.

(Dotted line in q: value of repairs included in N.)

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[1] *Cf.* S. FABRICANT, *Capital Consumption and Adjustment*, National Bureau of Economic Research, New York 1938, page 73.

[2] As a consequence of the "service-output method", as Fabricant calls it (*op. cit.*).

[3] Writings-off of capital losses are not included in the variable N. This is correct, since Z, at least for its principal purpose of explaining investment activity, should not take account of them either. There might be some influence of these writings-off on dividends, but no indication is found of its being important.

(5.10) "EXPLANATION" OF PROFIT FLUCTUATIONS

Profits play a central rôle in a society which is chiefly based on free enterprise. They will in many respects influence and determine the attitude of the entrepreneurs, and hence, indirectly, business activity and many other economic phenomena. It follows that, for our purpose (the explanation of real events), the definition of profits — which from the theoretical point of view is so ambiguous — has to be adapted as much as possible to the standpoint of entrepreneurs themselves, whether or not this yields a definition which is satisfactory from any normative standpoint. The equation "explaining"[1] profits should therefore be a picture of the calculations which the representative entrepreneur makes in order to find his profits. For this purpose, all enterprises have been combined into two groups, viz.: (i) those producing durable capital goods and their raw materials and semi-finished intermediate goods, and (ii) those producing other goods and services. For both groups, profits are the difference between receipts and total deductions; total profits are the sum of the two group figures.[2]

Receipts are assumed to consist of the value of goods and services sold, since such items as inter-business payments of interest, rents and dividends cancel out for all industries together. Sales are composed of home sales and exports. For the two groups, their sum will be equal to $U + V + U^e$.[3]

Deductions are assumed to consist of :

Total wages and salaries $(L_w + L_s)$
Managers' salaries (L_c)

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[1] In a sense, this equation could be called a definition equation, which would belong rather to Chapter I. But it is of course indifferent in which chapter each equation is discussed.

[2] A separate treatment for the two groups of enterprises seems hardly necessary. First, there is a striking parallelism between the two profit series, even after 1932; and, secondly, this separate treatment would be useful only if investment figures for these two groups separately were also known, which is not the case.

[3] One might perhaps have expected U' (home sales) instead of U (production for home market) in this formula. But when, *e.g.*, sales are lower than production, investment in stocks takes place, and the wages paid should therefore not be counted as costs for current sales. As we take in (5.10) all wages paid as costs, we must also take total production and not total sales.

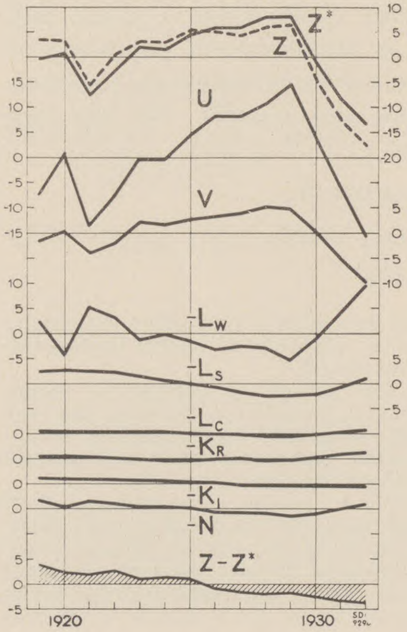
Net rents	(K_R)
Net interest	(K_I)
Depreciation allowances	(N)
Imports	(U^i)

Raw-material costs other than for imported raw materials, and home sales of unfinished goods are not to be included, as they cancel out within the national economy. On the other hand, all imports are to be considered as raw materials, since retail trade, etc., is included in our groups and virtually nothing will be imported directly by the ultimate consumer.

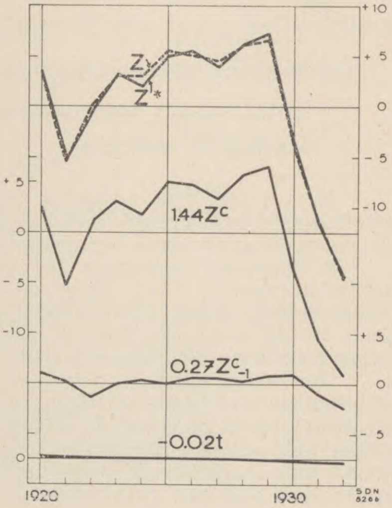
Thus, the following relation is found:

$$Z = U + V + U^e - U^i - (L_w + L_s + L_c + K_R + K_I + N) \quad (5.10).$$

Graph 5.10.
"Explanation" of Fluctuations in PROFITS.



Graph 5.11.
Relation between Fluctuations in TOTAL PROFITS and in CORPORATION PROFITS.



As all coefficients in this relation have values that are *a priori* equal to 1, statistical testing is extremely simple. It consists only in confronting calculated values of Z with actual ones. This has been done in graph 5.10, from which it will be seen that the chief difference is a trend difference.[1] In addition, there is a difference of nearly 5 milliard dollars in average level for which no explanation has yet been found; it must probably be ascribed to inexactitudes in average levels of other items. For the purpose of this study, this is of no importance, and the test can therefore be said to be favourable.

(5.11) "EXPLANATION" OF THE RELATION BETWEEN TOTAL PROFITS AND CORPORATION PROFITS

The profit series used as an explanatory variable has always been corporation profits; sometimes because they actually are the influencing factor; at other times because they are more accurately known than general profits and are probably a good indication of them. This latter fact has been tested in relation (5.11) where it is actually found that the two variables move very nearly parallel, but with a difference in (absolute)[2] amplitude, a small lag of general profits behind corporation profits, and a trend difference, representing the growth of the corporation form of enterprise. The relation runs:

$$Z = 1.45\,Z^c + 0.26\,Z^c_{-1} - 0.02\,t. \qquad (5.11).$$

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[1] The difference $U^e - U^i$, being very small, has been neglected.

[2] The percentage fluctuations of corporation profits are about twice as large as those of all profits.

CHAPTER VI

POSITIVE CONCLUSIONS ABOUT CYCLICAL MOVEMENTS IN THE UNITED STATES, 1919-1932

(6.1) CONCLUSIONS ON "DIRECT" RELATIONS

The system of relations established permits of a considerable number of conclusions about the actual course of events in the United States between 1919 and 1932.

A rather elementary way of reaching conclusions is simply to consider the graphs representing the result of each correlation calculation made. In this way it may be seen, for each year and each variable, in what proportions the various causes of changes — as far as they have been considered — have contributed to these changes. Some examples may be given.

Equation (5.10) shows the relative strength of the various components in the combined profit calculations of all entrepreneurs. Considering movements from 1928 to 1929, it appears that the value of consumers' goods production was still increasing, whereas that of producers' goods production was already decreasing. In the same interval, wages were increasing, tending to decrease profits.

Graph 1.13 shows that the decrease in value of investment goods in those same years is wholly due to residential building and not to other investment. Further, graph 2.5 indicates the causes of the decline in residential building. The number of houses some four years before was very high, and this discouraged building in 1929.

Taking the fall in general investment from 1929 to 1930 — which contributed considerably, according to graph 5.10, to the fall in profits in 1930 — we find from graph 2.4 that profits one half-year before were the chief explanatory series. Here we meet a very important feature. It would seem as if this were a circular reasoning: profits fell because investment fell, and investment fell because profits fell. This is, however, an inexact statement. Profits in period t fell because investment in period t fell, but the latter fell because of a fall in profits in period $t_{-\frac{1}{2}}$; and owing to this time lag there is no danger of circular reasoning.[1] Moreover, this lag is important in that changes in it may change considerably the resulting movements of both series, as will be shown below.

Let us go back to the fall in profits in 1930 and study the influence of consumption U' as affecting production of consumers' goods U. This fell considerably, and the fall in costs which accompanied it was not able to compensate it. Relation (2.1) tells us that one of the proximate causes of the fall in consumption was a decline in wages and in other consumptive expenditure. The result of the fall in wages is, however, almost entirely counterbalanced in Z by the rôle of wages as costs. Graph 2.1 gives also the proximate causes of the fall in non-workers' consumption. Here we find that a fall in capital gains had already caused a decline in consumption out of capital gains [2] between 1928 and 1929. Consumption out of other income was still rising. Capital gains fell because the *rate of increase* in stock prices, upon which, of course, they depend, falls before stock prices themselves fall. Here, a sort of "acceleration principle", but of an economic significance quite different from the ordinary acceleration principle, has an important influence.

Taking graph 1.3, we find a remarkable divergence between the various income types; it appears that dividends D, especially, remained high in 1930, and interest income K_I remained high all through the depression. Entrepreneurial withdrawals — corresponding to profits in non-corporate enterprises — fell heavily.

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[1] Even without lag it is possible to avoid circular reasoning, but the argument would be somewhat more complicated. Investment activity and profits would then both be determined by other variables.

[2] This is shown by figures for the sales of expensive motor-cars.

The foregoing conclusions are examples of the type of information obtainable for one special turning-point. The graphs may also be used in a slightly different way — viz., in order to obtain a number of statements valid for the period studied as a whole. This may, even more than the analysis of a single turning-point, give an impression of the forces which are most important in the business-cycle mechanism as a whole. The following is an attempt to formulate some of these statements.

The fluctuations in total value of production of both consumers' goods and investment goods have been caused much more by quantity fluctuations than by price fluctuations (*cf.* graphs 1.10, 1.15 and 1.16). The fluctuations in total profits, which are chiefly caused by fluctuations in total sales (*cf.* graph 5.10) have therefore also been chiefly governed by quantity fluctuations; only in a closer approximation are prices important. Clearly, an exception must be made for agricultural raw materials, where the reverse is true; their proportion in total production is, however, restricted to about 10%.

The influence on investment activity of what are usually considered as the most important "brakes" on an expansion — viz., interest rates and other costs — seems to have been very moderate (*cf.* graph 2.4). This is due not so much to the moderate size of fluctuations in interest rates and prices, as to the low elasticities.

Consumption outlay depends on two types of income, which are governed by rather different laws. Wages, salaries, dividends, rent and interest payments lag more or less behind general profits, whereas capital gains, by their very nature, lead (*cf.* E and G, graph 2.1).

The monetary sphere seems to be much less narrowly in contact with the physical sphere than one might expect. A superficial inspection of the graphs shows that the fluctuations in interest rates do not correlate narrowly with those in general production. The shape of the waves is clearly different for both groups. Equation (4.422) and graph 4.92 suggest that fluctuations in gold stock are a very important factor influencing interest rates; graph 4.63 suggests in addition that, the supply of money being much more elastic than the demand for it, the fluctuations in gold stock will hardly be found in total money in circulation. Apart from the influence of gold movements, there is an influence of general activity — productive as well as speculative — on interest rates. As already stated, however, the influence of interest rates on production and speculation seems to have been minimal (*cf.* sections (2.4), (2.5), (4.3) and (4.7). It must not be forgotten, of course, that these conclusions cannot be generalised for any business-cycle period in any country; to some extent they seem, on the contrary, to be very specific.

(6.2) CONCLUSIONS ON INDIRECT RELATIONS; THE ELIMINATION PROCESS AND THE "FINAL EQUATION"

The rather elementary types of conclusion given above, which deal with one equation at a time, and hence with proximate causes only, are for that very reason somewhat superficial. The method used is not expedient, either for arriving at a picture of the course of business cycles as a whole, or for considering the consequences of economic policy. To attain the first object, starting for instance with the fall in profits after 1929, we should have to pass in endless procession from one equation to another, to find more and more remote causes. On the other hand, when studying, say, the consequence of a sudden lowering, in 1929, of wage rates by 10%, one cannot of course simply deduce that profits would have been increased by 10% of the wage sum L_w, and stop at that. A change in wage rates changes prices (3.5) and production (2.4); it changes consumption (2.1) and thereby production and. . . wage rates (3.1). Here, again, we would have to follow the effects through all equations, but now in the opposite direction.

For both purposes it is therefore necessary to have recourse to another method. The general characteristics of the business cycle may, as it is exposed in the Introduction, be found by the elimination process, which will now be taken up. Problems of policy will be dealt with in section (6.8).

In principle, we shall now try to eliminate all variables but one from our equations, and to obtain one equation, to be called the "final equation", in which only one of the variables — say Z^c — will appear together with a number of data. This elimination process is very laborious, and can in fact only be carried out with the help of further simplifications. According to whether more or fewer of these are adopted, we may obtain a rough first approximation or more refined second, third, etc., approximations. The latter are, of course, more exact, but far more complicated; for reasons of clearness it will therefore often be more helpful to take the less exact formulæ.

In the elimination process, all trend terms will, from the start, be disregarded. This does not involve any special simplification, but simply means that our results are obtained not for the variables as they stand, but for the deviations they show from some straight line in time (a different one for each variable).[1] This straight line will be considered as a structural development, in which we are for the moment not interested.

Further, all terms containing cumulants, like $\int Z$, will be omitted, since some calculations have shown that they have no large influence on the shape of the shorter fluctuations.[2]
found for one variable may afterwards be transformed for another variable.

The exact course of the elimination process is largely dependent on the mathematician's choice. In principle, he may start where he likes and may eliminate variables in what order he likes. He may also freely choose what variable or variables he likes to keep in his final result. This does not matter very much, at least in principle, since any result found for one variable may afterwards be transformed for another variable.

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[1] This straight line need not be the rectilinear trend of each series. It would be so if we had introduced a trend in every equation. For then, owing to a well-known theorem of multiple correlation analysis (proved by Frisch and Waugh), the regression coefficients would have been the same as if beforehand each variable had been replaced by its deviations from trend.

[2] A more exact argumentation can only be given at a further stage. *Cf.* pages 147 *sqq.*

Here, the extremely simple example of the Introduction:

$$V_t = \beta Z_{t-1} \qquad (6.21)$$

$$U_t = L_t + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2}) \qquad (6.22)$$

$$Z_t = U_t + V_t - L_t \qquad (6.23)$$

may be reconsidered.

Z_t may be kept by eliminating V_t and $U_t - L_t$[1] by substituting (6.21) and (6.22) in (6.23):

$$Z_t = \beta Z_{t-1} + \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2})$$

or:

$$Z_t - (\beta + \varepsilon_1 + \varepsilon_2) Z_{t-1} + \varepsilon_2 Z_{t-2} = 0 \qquad (6.24).$$

It is also possible to keep V_t by first solving (6.21) for Z_{t-1}:

$$Z_{t-1} = \frac{1}{\beta} V_t, \qquad (6.25)$$

from which it follows that:

$$Z_{t-2} = \frac{1}{\beta} V_{t-1} \text{ and } Z_t = \frac{1}{\beta} V_{t+1} \qquad (6.25').$$

In addition, (6.23) must be solved for $U_t - L_t$:

$$U_t - L_t = Z_t - V_t = \frac{V_{t+1}}{\beta} - V_t \qquad (6.26)$$

and the result substituted in (6.22):

$$U_t - L_t = \frac{V_{t+1}}{\beta} - V_t = \varepsilon_1 Z_{t-1} + \varepsilon_2 (Z_{t-1} - Z_{t-2})$$

$$= \frac{\varepsilon_1}{\beta} V_t + \frac{\varepsilon_2}{\beta} (V_t - V_{t-1})$$

or:

$$\frac{V_{t+1}}{\beta} - \frac{\beta + \varepsilon_1 + \varepsilon_2}{\beta} V_t - \frac{\varepsilon_2}{\beta} V_{t-1} = 0 \qquad (6.27).$$

---

[1] It will be seen that $U_t - L_t$ must and can be considered as a single variable in these cases.

It will readily be seen that this equation for V is the same as (6.24) for Z; the only differences being that (6.27) has been divided by β and relates to one time unit later than (6.24).

Nevertheless, in practice it sometimes makes a good deal of difference where one starts, and the particular structure of the equations may very much facilitate some course. On closer examination of the system (*cf.* Appendix B, table I) one finds that the equations may be ranged in four groups. First there is the group of monetary equations, which may, by elimination, easily be reduced to only two equations, expressing n and m_{Lb} in non-monetary variables. Secondly, there is a group of equations which may immediately be substituted in the others, each reducing thereby the number of equations and of the variables by one. Equations (1.2), (1.3), (1.7), (1.9), (1.13), (1.14), (1.15), (1.16), (2.2), (5.1), (5.2), (5.3), (5.4), (5.5), (5.6), (5.7), (5.9) and (5.11) belong to this group. After the substitution, there remains the set given in table III, which may now be subdivided in two groups:

> (i) A "price" group, containing equations (1.8′), (1.10′), (1.11′), (1.12′), (3.1′) to (3.5′) and (5.8′); and
>
> (ii) What for reasons to be mentioned later may be called the "strategic" group, containing the remaining equations (2.1′), (2.4′), (2.5′) and (1.4′), (2.6′), (4.6′), (4.91′) and (5.10′).

The structure is such that the first group consists of a number of relations "explaining" variables that play only a secondary role in the second group. The chief variables in the "strategic" group are: Z^c, corporation profits, n, share prices, v_B, residential building activity, v', other investment activity, and U', consumption. They may be called the "strategic variables". This grouping suggests the following treatment: the "price" variables may first be found as functions of the "strategic variables" and then be substituted in the "strategic" group of equations[1]. This has been done in tables

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[1] The substitution must necessarily be repeated when, after further elimination, some of the strategic variables are expressed in others.

IV and V, where the whole process is given step by step. We are then left with a kernel of relations which can more easily be treated. It is, of course, not by chance that we are left with these equations and these variables. The logical structure of our system of equations, which after all is nothing but a reflection of the structure of the business-cycle mechanism, is such that they play the central rôle. This is why we call them the strategic group for the understanding of the mechanism. Their coefficients will be seen to have the largest influence on the character of business cycles.[1] In order to simplify still further, we may, within the strategic group, eliminate the variable v', which is easily expressed as a function of the other variables. For reasons to be given fully in section (6.7), we consider v_B as an external variable. Greater difficulties arise if we try also to eliminate n and Z^c; for these variables do not occur only once in these equations, but several times, with various lags. This reflects the economic fact that these variables are connected by many causal chains, working in various directions and with various lags.

The expression of n in terms of the other variables is made especially difficult by the presence of the term $2.40\,(n - n_{-1} - 20)''$ and of H, which is equal to $0.3\,(Z^c_m - Z^c - 7)''$. The first term intends to indicate that in a speculative boom n is much affected by its own previous rate of increase. This has the result that n moves much by its own laws, pulling with it the other variables, as we shall be able to show. The H-term means that n is depressed by currency-hoarding in a severe depression. On account of these complications, it is useful to split up our considerations into three parts relating to the three forms of the equation for n:

> (i) Case I, the "normal interval", where $n - n_{-1}$ is less than 20 and has therefore no influence on n, and where Z^c either rises or does not fall so deeply that hoarding takes place;

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[1] *Cf.* section (6.9).

(ii) Case II, the "boom interval", where $n - n_{-1} > 20$ and consequently influences n, without hoarding;

(iii) Case III, the "depression interval", where $n - n_{-1} < 20$ and n is further kept down by the occurrence of hoarding. A fourth case, where a boom development of n would coincide with the depression phenomenon of hoarding, need not be taken into consideration.

The equation found for Z^c which still contains n-terms next to terms with Z^c and a number of variables that are taken as given for our system of equations, *e. g.*, f, runs:

$$\begin{aligned} 0.770\,Z^c = {} & 0.179\,Z^c_{-1} + 0.006\,Z^c_{-2} - 0.015\,Z^c_{-3} + 0.007\,Z^c_{-4} \\ & - 0.131\,h_{-2} - 0.083\,h_{-3} - 0.290\,h_{-4} - 0.845\,f \\ & + 0.335\,f_{-1} + 0.081\,v_B - 0.017\,(v_B)_{-1} + 0.090\,n \\ & - 0.049\,n_{-1} + 0.001\,n_{-2} + 0.003\,n_{-3} \qquad (6.28) \end{aligned}$$

(*cf.* Appendix B, table V, line 262 + 264).
In the next sections, we shall consider cases I, II and III for n.

Detailed Description of the Elimination Process.

The process starts in table II, where the equations of group 4 in table I are combined. Let us follow somewhat more closely the beginning of this process.

In line 1, equation (4.4) is copied, with the omission of the VC-term (the indication of the "explanatory" series is given at the top of each column to save space; for the same reason, the heading in one column is sometimes changed). In line 2, equation (4.63) is written, but transformed in an equation "explaining" M by applying equation (1.5) and adding M′ on both sides. Subtraction, in line 3, eliminates M; this equation may be written with m_S on the left-hand side (4); the factor -0.42 has immediately been applied since it is with this coefficient that m_S occurs in M (2). (2) + (4) gives again M (5), but now without m_S. If now, in all places where M or m_S occur, we replace them by the expressions (4) and (5) so found, these variables are eliminated from the system of equations (*cf.* lines 7 and 8). The same procedure is applied to other monetary variables until, in (21) and (22), m_{Lb} and n

are expressed in non-monetary or external variables (except H, which is kept for reasons of convenience).

The eliminations in table III are of a simpler type: certain variables in some of the equations, especially of the "strategic group", are simply replaced by the expressions by which they are "explained" or defined in some other equation. Thus, in (2.1), 0.77 E is first replaced by $0.77\,(D + L_c + \ldots)$ with the help of (1.3); subsequently 0.77 D is replaced by $0.77\,(0.151\,Z^c + \ldots)$ equation (5.1) and finally the S_{-1}-term so introduced is reduced to 0, according to equation (1.9) in its simplified form: all cumulants and trend terms omitted. The equations used are mentioned in the column "References".

In table IV, the procedure is again of a somewhat different type. The purpose of this table is to express the prices m_R, p^f, p, l, q, and the variables $u + v$ and L_w in "strategic" variables F^1, F^2 and $U + v$, where the F's represent certain expressions in the strategic variable Z^c and the external variables h and f. In line 101, $0.80\,p'$ in (1.8′) is substituted by $0.80\,(0.47\,l_{-0.21} + 0.25\,p^f_{-0.21})$. In the next line, the l-term in 101 is replaced by (3.1′). This gives an expression with p on the left-hand side and $0.147\,p_{-0.63}$ on the right-hand side. In line 103, these two terms are combined to $0.853\,p_{+0.11}$, where 0.11 is the weighted lag (lead) $+ 0.11 = [(1.000 \times 0) - (0.147 \times - 0.63)] \div 0.853$; then all terms of the equation are divided by 0.853 ($0.200 \div 0.853 = 0.234$, etc.) and shifted in time by -0.11 year. This procedure as a whole is indicated by the reference "R". This way of elimination is continued throughout the table. Attention should be drawn to the groups of terms taken from (3.4′) and (3.2′) respectively which are introduced as a whole in (110) and (111). Here, as well as in the subsequent introduction of F^1 and F^2, the procedure was dictated by considerations of simplicity and the avoidance of unnecessary calculations.

In table V, the different "strategic" variables are successively eliminated. In the first place U and U′ are treated. With the help of (2.1) and the results of the preceding table,[1] U′ may be replaced by $U + v$, F^1, F^2, F^3, where F^3 is a new

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[1] *Cf.* also note 1 on page 136.

combination, provisionally to be kept in this form. Equation (2.6′) gives another relation between U' and U (u' and u being transformable into U and U' via (1.12) and (1.10)). These two equations suffice to eliminate both variables, and to find U expressed in v and F's (216). It may be remarked that the same expression (206) is applied three times (in 207, 208, 209) but with different lags (0, 1 and 2 years).

In lines (217) to (224), v (or v', *cf.* (1.14)) is eliminated, and hence the result of (216) may be improved by expressing U without using v (225).

In (227) to (230), p^f is treated. The difficulty is here that F^3 contains $0.049\,\Delta p^f$, which causes the small p^f terms in the last column. But the latter are so small that, if they are replaced (229) by the expression for p^f in (228), the p^f terms they yield are no longer perceptible.

In (231) to (237) certain variables of tables IV and V are expressed in F^1, F^2, F^4, F^5 and v_B. Terms with lags of parts of a year are split into two terms with the same average lag.[1]

Certain combinations of these variables occur in n and Z^c (*cf.* (4.91′) and (5.10′)); their expression in F^1, etc., may now easily be found: S_n in (242) and S_z in (248).

The F's are then decomposed into terms with Z^c, n, h and f (249-252; 258-261). If we add the other terms in (4.91′) (253), we find, after a few transformations, n expressed in Z^c and external variables. We only need to substitute this expression for the n-terms in S_z to find Z^c expressed in values assumed by Z^c at moments lying 1, 2, 3 and 4 years back and in external variables (266).

(6.3) THE CHARACTER OF THE MOVEMENTS IN THE ABSENCE OF A STOCK-EXCHANGE BOOM AND OF HOARDING

To study case I, we omit both the term with $(n - n_{-1} - 20)''$ and that with H in line 257 (Appendix B) which explains n. We may now replace the n-terms in the Z^c-equation (*cf.* line

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[1] The same procedure is applied to small leads, *e.g.*, $-0.022F^1{}_{+0.23}$ is replaced by $-0.027F^1 + 0.005F^1{}_{-1}$, with the same average lag $(-0.022 \times +0.23 = +0.005 \times -1)$.

262) by an expression containing only Z^c at various moments and exogenous variables. In this way we get a "final equation" for Z^c, running:

$$\begin{aligned} 0.445Z^c = {} & 0.177Z^c_{-1} - 0.098Z^c_{-2} + 0.006Z^c_{-3} + 0.012Z^c_{-4} \\ & - 0.135h_{-2} - 0.077h_{-3} - 0.305h_{-4} + 0.74\,(\mathrm{A}u + \mathrm{P}) \\ & - 0.40\,(\mathrm{A}u + \mathrm{P})_{-1} - 0.822f + 0.315f_{-1} \qquad (6.30).^1 \end{aligned}$$

In order to facilitate the understanding of this equation and its consequences, it may be written in a somewhat more condensed form:

$$Z^c_t = e_1Z^c_{t-1} + e_2Z^c_{t-2} + e_3Z^c_{t-3} + e_4Z^c_{t-4} + (\mathrm{AU} + \mathrm{HO} + \mathrm{F} + \mathrm{R})_t \qquad (6.31).^2$$

Here e_1 to e_4 are numbers depending, in principle, on almost all regression coefficients in all elementary equations. They describe in an abbreviated form the structure of the economic mechanism with regard to business cycles; they will be different in other countries, or under another regime, where the economic structure of society is different.

The other four new symbols have this in common, that they may be considered as *largely independent* of the general business-cycle position. Their exact meaning may be discussed later. This co-existence in formula (6.31) of two types of terms — independent terms and terms depending on previous values of Z^c — is of importance. It represents the fact that at any moment profits Z^c (and quite similar propositions hold, as we have already indicated, for the other variables) are the product of two types of forces: forces connected with previous business-cycle situations $(e_1Z^c_{t-1} + e_2Z^c_{t-2} + e_3Z^c_{t-3} + e_4Z^c_{t-4})$, and independent forces which are often indicated as *disturbances*, since their changes cause Z^c not to follow the regular pattern of cycles. They are also indicated as *external* or *extraneous forces* (which indicates their origin; for some of them this expression is more appropriate than for others, as we shall see), or *shocks* (which of course bears

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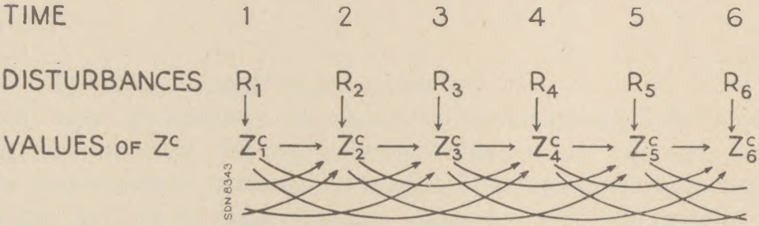
[1] *Cf.* Appendix B, table V, line 265.

[2] The meaning of the last four symbols will be described in the next pages.

somewhat more upon the possibility of their sudden changing and is not therefore equally applicable to all of them), or *starters* (which reminds of the possibility — especially if they come in a rather quiet period — that they may be the beginning of a new cyclic movement).[1] The causal connections which are described in equation (6.31) may be illustrated by the following diagram, where one symbol R stands instead of the sum AU + HO + F + R:

Graph 6.31.

Causal Connections between
Disturbances and Profits



The arrows indicate causal connections. Each value of Z^c depends immediately on certain disturbances, but it depends also on the earlier ones through its connection with the Z^c-values for one, two, three and four[2] years back.

Equation (6.31) tells more than this very general statement. It tells what is the origin of the first three disturbances AU, HO and F. Going back to the equations that describe these three variables, we will even be able to determine the magnitude of the disturbances in any year.

AU represents influences coming from changes in the gold stock and in the autonomous component P in central banking policy; it is

$$AU_t = 1.66\,(Au + P)_t - 0.90\,(Au + P)_{t-1} \qquad (6.32).$$

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[1] Dr. Johan Åkerman considers as the "real causes" of the business cycle these external forces; our own preference being to indicate by that term the structure of the economy, represented by the coefficients e_1, e_2, e_3 and e_4. It is, of course, only a question of definition.

[2] In order not to overload the diagram, the arrows from Z^c_1 to Z^c_5, Z^c_2 to Z^c_6, etc., corresponding with the term $e_4\, Z^c_{t-4}$ have not been drawn.

The isolation of these terms seems especially interesting in judging the influence of banking policy and gold movements — past as well as potential.

HO represents influences coming entirely from the housing market and, more exactly, from a development in the housing market that is largely a product of events more than three years back (and therefore, as already observed, in a very high degree independent of Z^c_{t-1}, Z^c_{t-2}, which are most important in the final equation). It is given by the formula:[1]

$$\mathrm{HO}_t = -0.303h_{t-2} - 0.173h_{t-3} - 0.685h_{t-4} \qquad (6.33)$$

and depends on the number of houses in existence two to four years before. Through its large influence on the actual building volume, this number acts also on the present value of Z^c. The usefulness of taking it as a separate item is (i) that in no other part of the economic system were such large lags found to have a considerable influence on the cyclic movements;[2] (ii) that it shows almost autonomous cycles, to be discussed later, and (iii), that, for that reason, we are able to evaluate the influence of housing on the general business situation.

F stands for the influences, chiefly climatic, which change crops; they are generally accepted as important external forces. Again it seemed useful to isolate these terms.

$$\mathrm{F}_t = -1.847f_t + 0.708f_{t-1} \qquad (6.34).$$

R, finally, is an agglomerate of a non-discernible multitude of disturbances which, each in itself, seem far less important than the three types mentioned, but taken together may still be important. Because of their large number and, in all probability, mutual independence, they may, however, be treated as random disturbances.

Although we have succeeded in giving separate terms for at least some of the most important external factors, there are two categories which may also be important and have not been

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[1] *Cf.* Appendix B, table V, line 266.

[2] One could have expected that the so-called "echo principle" would also give an example of such forces, and wonder why it has no place in this system of equations. Very probably, however, these forces are of importance only for the explanation of trend movements. *Cf.* Vol. I, Chapter III, and section (2.1) of this volume.

included, viz., inventions and, in the United States especially for the period after 1932, Government policy. The latter, if well devised, will, however, belong rather to the class of regular exogenous factors such as the terms HO in our example.

From the business-cycle point of view the first four terms in equation (6.31) are the more interesting. They represent the systematic cyclical forces. They tell us that, apart from disturbances, the situation of to-day will depend on the situations of one, two, three and four years ago; and — if the problem is studied more accurately — even of a number of more remote years; the influence of the latter is, however, found to be small.

Looking for a moment at this systematic part only, we get the relation

$$Z_t^c = e_1 Z_{t-1}^c + e_2 Z_{t-2}^c + e_3 Z_{t-3}^c + e_4 Z_{t-4}^c, \qquad (6.35)$$

which is called a "difference equation". It enables us to calculate the future movements (in the absence of new disturbances) if there are given:

(i) four initial values, say Z_{1917}, Z_{1918}, Z_{1919} and Z_{1920}, and

(ii) four coefficients e_1, e_2, e_3 and e_4, which depend on the coefficients in our elementary equations and therefore in the widest sense upon the economic structure.

In our example, the period and the damping degree of the endogenous movements depend only on e_1 to e_4 (*i.e.*, on the structure), whereas the amplitude depends on the initial values, say Z_0^c, Z_1^c, Z_2^c and Z_3^c. In more complicated cases these influences may be mixed up in various ways. If the endogenous movement is damped, it will vanish after some time and a new movement will develop only if fresh disturbances occur.

The numerical values for the coefficients in our case lead to the following formula:

$$Z_t^c = 0.398 Z_{t-1}^c - 0.220 Z_{t-2}^c + 0.013 Z_{t-3}^c + 0.027 Z_{-4}^c \qquad (6.36).$$

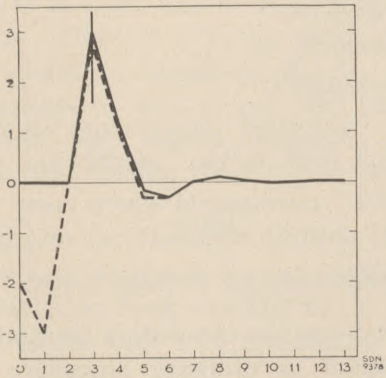
Choosing some arbitrary initial values, *e.g.* $Z_0^c = 0$, $Z_1^c = 0$, $Z_2^c = 0$, $Z_3^c = 3$, the further development may be calculated. It is given in graph 6.32, and consists chiefly of a damped cycle with a period of 4.8 years. It may be proved mathematically that this is the case independently of the initial values of Z^c chosen;[1] this statement could be tested by some trials with other values. This cycle is somewhat longer than the well-known "short American cycle", of which the average length has been estimated at forty months. Neither of the figures should, however, be taken too literally; for, on the one hand, the average length of cycles just quoted is based upon measurements of actual cycles which are always subject to disturbances, and, on the other hand, our result too is subject to a considerable margin of error.[2] It may be shown, however, that the other conclusions — those regarding the influence of policy and of external factors — are far more certain.[3]

Graph 6.32.

ENDOGENOUS MOVEMENTS OF Z^c (within the "normal interval" for share prices and in the absence of hoarding) FOR TWO SETS OF INITIAL VALUES. Time in Years.

——— Initial values: $Z^c_0=0$, $Z^c_1=0$, $Z^c_2=3$.

- - - - Initial values: $Z^c_0=-3$, $Z^c_1=0$, $Z^c_2=3$.



The mechanism may be made somewhat more understandable by the following analysis: Equation (6.36) shows two large forces acting on Z^c_t: first, a force in the same direction as Z^c_{t-1};

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[1] *Cf.* below.

[2] *Cf.* page 179.

[3] *Cf.* J. TINBERGEN: "On the Theory of Business-cycle Control", *Econometrica* VI (1938), page 22.

secondly, one in the direction opposite to Z^c_{t-2}. If now Z^c_{t-1} and Z^c_{t-2} have the same sign—*i.e.*, if the system finds itself either distinctly above normal or distinctly below normal—the positive and negative forces will counteract each other and the new Z^c will be small, *i.e.*, profits will be nearer to normal. This, in a nutshell, is the turning-point situation. If, on the other hand, Z^c_{t-1} has a sign different from that of Z^c_{t-2}, the forces will reinforce each other and the new Z^c_t will be of the same sign as Z^c_{t-1}. If the absolute value of Z^c_{t-1} is small in comparison with Z^c_{t-2} — *i.e.*, if the normal position is only just passed — Z^c_t will be larger than Z^c_{t-1}: this is the cumulative process. For a good understanding of this result, as well as of some of our further statements, it may be observed that even in the simple case with only two coefficients, $Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2}$, the connection between (i) the coefficients e_1 and e_2, and (ii) the character of the endogenous movement is not so simple as one might expect. The following types of movement are possible:

(*a*) Cyclic movements, with any period of at least two years, either
  (i) damped, or
  (ii) undamped, or
  (iii) anti-damped;

(*b*) Damped "one-sided" movements which gradually carry the system back to the "equilibrium position" $Z^c = 0$;

(*c*) "Explosive" movements which carry the system away from that position without ever returning to it;

(*d*) Several combinations of these types.

The rules indicating the connection mentioned above cannot easily be formulated in ordinary language; a mathematical formulation is the only one possible. The problem is to find a function of time,[1] say Z^c_t, which identically satisfies the equation

$$Z^c_t = e_1 Z^c_{t-1} + e_2 Z^c_{t-2} \qquad (6.371).$$

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[1] Throughout the following pages, the variable time is supposed to assume entire values $t = \ldots 0, 1, 2 \ldots$ only, the corresponding values of Z^c_t being conceived as annual averages of profits for the year t.

The mathematician finds this function by trial, and proves afterwards that the solution found is the general one. A function[1]

$$Z_t^c \equiv Kx^t, \tag{6.372}$$

where K and x are constant, is tried. The substitution of this function in (6.371) gives

$$Kx^t = Ke_1x^{t-1} + Ke_2x^{t-2}.$$

From this it is clear that K may be chosen arbitrarily, since each value for K will be correct once x has been chosen so as to satisfy $x^t = e_1x^{t-1} + e_2x^{t-2}$, or

$$x^2 - e_1x - e_2 = 0 \tag{6.373}.$$

This quadratic equation, formed with the coefficients of the equation (6.371), is called the characteristic equation. Its roots x_1 and x_2:

$$x_1 = \frac{1}{2}e_1 + \sqrt{\frac{1}{4}e_1^2 + e_2}, \quad x_2 = \frac{1}{2}e_1 - \sqrt{\frac{1}{4}e_1^2 + e_2}, \tag{6.374}$$

are the only values of x for which the function (6.372) satisfies (6.371). All other values of x would yield no solution at all, whatever values might be chosen for K.

As the coefficients e_1 and e_2 are real numbers, the roots x_1 and x_2 are either both real or conjugate complex. If x_1 is real and positive, but smaller than 1, the curve

$$Z_t^c = Kx_1^t \tag{6.375}$$

represents a gradual approach to an equilibrium situation ($Z^c = 0$). The deviation from equilibrium in the year $t + 1$ is found from that in the year t by dividing it by the factor

$$D = \frac{x_1^t}{x_1^{t+1}} = \frac{1}{x_1} \tag{6.376}$$

which may be called the "damping ratio" of the movement. The larger D, the faster the equilibrium is approached. If x_1

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[1] The sign $\equiv$ indicates that the equality of Z_t^c and Kx^t is meant to be true for every value of t.

is larger than 1, however, the "damping ratio" is smaller than 1, and the movement leads farther and farther away from the equilibrium situation (upwards or downwards according to the sign of K).[1]

If both real roots x_1 and x_2 are positive, the movement

$$Z_t^c = K_1 x_1{}^t + K_2 x_2{}^t, \tag{6.377}$$

which is a combination of two solutions, each of one of the above types, also constitutes a solution. Substitution of the expression (6.377) in (6.371) shows this at once. The arbitrariness of the constants K_1 and K_2 makes it possible to choose them in such a way as to give prescribed values to Z_t^c for two points of time — *e.g.*, $t = 0$ and $t = 1$. This only requires that K_1 and K_2 should be determined by the two equations:

$$\begin{aligned} K_1 x_1{}^0 + K_2 x_2{}^0 &= Z_0^c \\ K_1 x_1{}^1 + K_2 x_2{}^1 &= Z_1^c \end{aligned}$$

which, since $x_1{}^0 = x_2{}^0 = 1$, reduces to:

$$\left.\begin{aligned} K_1 + K_2 &= Z_0^c \\ K_1 x_1 + K_2 x_2 &= Z_1^c, \end{aligned}\right\} \tag{6.378}$$

where Z_0^c and Z_1^c are the prescribed values of Z_t^c for $t = 0$ and $t = 1$.

Thus it will be possible to find one determinate solution for each given pair of values for Z_0^c and Z_1^c.[2]

If the roots x_1 and x_2 are conjugate complex, it may be shown that the expression (6.377) is equivalent to

$$Z_t^c = K a^t \sin \frac{2\pi}{T}(t - \tau) \tag{6.379}.$$

If the factor a^t were absent from this expression, Z^c would move in an undamped harmonic oscillation with a period of T years. The factor a^t brings about a gradual increase or decrease

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[1] $y = Kx^t$ also represents the value in the year t of a capital K, invested in the year $t = 0$ at compound interest at the annual rate of $100\,(x - 1)$ per cent.

[2] It is possible to prove that no other solutions exist.

n the amplitude of this oscillation, according as $a > 1$ or < 1. Calling, again,

$$D = 1/a$$

the damping ratio, the oscillation is damped if $D > 1$, and anti-damped if $D < 1$.

The period [1] T and damping ratio D of the solution (6.379) depend as follows on the coefficients of (6.371):

$$\tan \frac{2\pi}{T} = \frac{x_1 - x_2}{x_1 + x_2} = \sqrt{1 + 4\frac{e_2}{e_1}} \qquad (6.380)$$

$$D = \frac{1}{\sqrt{x_1 x_2}} = \frac{1}{\sqrt{e_2}} \qquad (6.381).$$

On the other hand, the initial amplitude K and phase τ of the oscillation are not prescribed by the coefficients in (6.371), but depend only on the initial values Z_0^c and Z_1^c of Z^c. They may be found in the same way as K_1 and K_2 were found in (6.378).

From (6.380) it follows that a value above two years may always be found for the period T. A limiting case is formed by a negative real root x_1, which leads to an oscillation of an exact two-year period, with a damping ratio $D = \frac{1}{|x_1|}$ and an initial amplitude determined by the initial values of Z^c.

In the case of the simple model quoted in the Introduction, we have (*cf.* equation (0.4), page 17) $e_1 = +1.6$, $e_2 = -1.0$, and therefore $x^2 - 1.6x + 1 = 0$; the roots x_1 and x_2 are: $0.8 \pm \sqrt{0.64 - 1}$ or $0.8 \pm 0.6i$; the corresponding movements are cyclic and undamped, and show a period of ten time units of four months. This number is found as the quotient of 360° (or 2π radiants) by arc $\tan \frac{0.6}{0.8}$.

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[1] Though equation (6.380) admits of an infinite series of solutions T_k satisfying $\frac{2\pi}{T_k} = \frac{2\pi}{T_0} + k\pi$, $k = 0, \pm 1, \pm 2 \ldots$, with $0 < \frac{2\pi}{T_0} \leq \pi$, the restriction of t to entire values (see note 1 on page 142) makes the expression (6.379) for each of these solutions mathematically equivalent to that corresponding to the value T_0.

No other types of movement than those described above occur when the final equation, unlike (6.371), contains also terms with Z^c_{t-3}, Z^c_{t-4}, etc. Every additional term, however, increases by 1 the degree of the characteristic equation, and with that the number of its roots. Thus, the general solution of a final equation containing terms up to Z^c_{t-4} is a combination of four specific movements, each of one of the above types, and each corresponding to one root of the characteristic equation. Though in general the initial relative importance of the specific movements is determined by the four initial values of Z^c, the specific movements most important for business-cycle analysis are those which have the smallest damping ratio, as such movements are most likely to persist for a longer time.

In the present case, the final equation (6.30) contains as the most important movement a cycle with a period of 4.8 years and a damping ratio of 1.89.

The period and damping ratio depend, in principle, on each coefficient in each single equation which has been used in deriving the final equation. In section (6.9), the influence of changes in some of the more important coefficients on damping and period are studied, and some indication is given of the uncertainty in the above figures ensuing from given margins of error in those coefficients. One instance, however, is sufficiently interesting to be mentioned here, as it indicates a probable bias in the above figures.

Owing to the large damping ratio, there is a tendency for damping and period to be particularly sensitive to changes in coefficients in the elementary equations that represent causal connections acting with a large time-lag. The only case of a lag above one year in the elementary equations (leaving aside the term with h_{-4} which is considered as an external variable) is in equation (5.7), and it appears that, if (5.7) is replaced by (5.7′) — see page 120 — the period works out at 5.0 years and the damping ratio at 1.59. As the fit in the explanation of L_s is nearly equally good in the two alternatives, it would seem that the choice has to depend on other considerations. Damping and period for any set of coefficients in the L_s-equation intermediate to the extreme cases (5.7) and (5.7′) may be found approximately by linear interpolation between the figures given.

For most problems to be studied in the following sections, the choice between the two possibilities is not important; to save work, only the final equation based on (5.7) has been applied in such cases. Only in the second half of section (6.9) some figures are given on the basis of both equations for L_s.

According as more terms occur in the final equation, the computation of the damping ratios and periods of its specific solutions becomes more laborious. Now that the principles have been explained for the simplified case (6.371), we shall make only two general inferences which are of great importance for business-cycle theory and business-cycle policy.

(i) The damping and period, and even the type, of the possible movements defined by the final equation depend very much on the numerical values of its coefficients.

(ii) The effect of certain measures of business-cycle policy on the stability of the economic system may be studied by means of the effect of such measures on the coefficients in the final equation. Stability will be promoted by measures that increase the damping ratio of the solutions to that equation.[1]

The omission of the cumulants in the elimination process may now be explained in somewhat more detail.

In a preliminary calculation, these terms were retained throughout the elimination process. The resulting occurrence of terms containing cumulants in the final equation influences in two ways the possible movements of the system:

(i) The period and degree of damping of the cyclical movement are to some extent affected by the presence of such terms.

(ii) Besides that, the cumulants introduce an additional root into the characteristic equation, which is real and positive, giving rise to a one-sided movement. This movement is explosive (away from the equilibrium situation) if the algebraic sum of all coefficients of cumulation terms in the final equation is positive; the movement is damped (gradual approach of the equilibrium situation) if that sum is negative.

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[1] All this may be formulated independently of where the equilibrium lies. It is, however, quite possible that the desirability of a high equilibrium level might conflict with the requirement of a "very stable" or a "strongly damped" movement. In a private publication this problem has been studied somewhat more closely: *cf.* J. Tinbergen, *Fondements mathématiques de la stabilisation des affaires* (Hermann, Paris, 1938).

The upshot of the preliminary calculations was that the $\int Z^c$-terms in the final equations, which resulted from the cumulants taken into consideration in the elementary equations, (i) had a small negative influence (about — 0.05) on the damping ratio and a small positive influence on the period of the cyclical solution (about 0.5 year); (ii) gave rise to a solution corresponding to an explosive movement (root $x_5 = 1.14$).

If such a solution really constitutes a possibility inherent to the economic system of the United States, simple calculations — for instance, numerical solutions as given in sections (6.5), (6.6) and (6.7) — show that, granted certain initial conditions, the one-sided type of movement could lead, after a few years, to values for the variables so far from the average, and, consequently, so far beyond the range in which the elementary equations have been determined, that the latter, and hence the final equation, could no longer be assumed to hold. This exponential solution, if it were kept to, would have to be interpreted as the expression of a cumulative process to which a special explanation of the turning-points, by bottle-necks and other non-linearities in the equations, would have to be added.

This last result seems unsatisfactory in two respects. First, though the conception of a cycle as consisting of two alternating cumulative processes, an upswing and a downswing, linked by two turning-points, is not uncommon in general cycle theories,[1] in few of these theories are the cumulative processes attributed to those phenomena that are represented by the $\int Z^c$-terms in the final equation — the surplus of corporations (equation (1.9)), working through its influence on dividends (equation (5.1)), the stock of capital goods through its influence on depreciation (equation (5.9)), and the amount of long-term debt outstanding, working through interest payments (equation (5.4)). An explanation of the cycle that would be largely dependent on the growth of these three assets may therefore be rejected *a priori* on theoretical grounds.

The second objection is that the cumulants in the elementary equations that have led to this explosive exponential

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[1] *Cf.* G. Haberler, *Prosperity and Depression.*

root are not all, and perhaps not even the most important of, the cumulants to which the economic mechanism gives rise in reality. At many places, trends have been used which properly represented cumulations of physical quantities or values. Of the cumulant, for example, of physical investment,

$$\int \bar{\bar{v}} = \int \bar{v} + \int v,$$

the first component moves very gradually, and the influence of $\int \bar{\bar{v}}$ on any other variable will be statistically distinguishable, in the sense of the correlation technique, from that of other gradually operating factors only if the oscillations in the much smaller term $\int \bar{v}$ render $\int \bar{\bar{v}}$ different from a smooth curve — which is not markedly the case for most cumulants (see graphs 1.9, 5.4, 5.9). Thus in a number of cases an accurate determination of the coefficients of cumulants was not possible, or not important since it was impossible in so many other cases (equations (1.2), (4.1) and (4.2), and their use in (4.82), (4.63), (5.2), and perhaps also the trends in prices: (3.3), (3.4), (3.5)).

A rough estimate of the possible effects of these "hidden" cumulants in the elementary equations showed (i) that their influence on the cyclical solution could not be very large, and would change the damping factor by a figure for which ± 0.05 could be set as an extreme limit; (ii) that the sign of the algebraic sum of the coefficients for the cumulants in the final equation, including terms resulting from the "hidden" cumulants, could not be determined without a more precise knowledge of the coefficients of these hidden cumulants than seems attainable at present. On the basis of the information available, not much more could be said than that the positive real root probably lay somewhere between 0.75 and 1.25, which leaves the possibility of either a damped or an explosive one-sided movement. The latter has to be rejected for reasons already given above; the influence of the former on the cyclical movements would be moderate and therefore in accordance with the movements observed in reality. To sum up, on the ground of their small influence under (i) and our ignorance of their effect under (ii), it seemed both advisable and justified to keep all terms containing cumulants out of the elimination process.

(6.4) AN ECONOMIC INTERPRETATION OF THE FINAL EQUATION IN THE ABSENCE OF A STOCK-EXCHANGE BOOM AND OF HOARDING

The final equation for Z^c discussed in section (6.3) may be interpreted economically. This interpretation clearly holds only for the non-speculative interval, as outside that interval the coefficient of the *n*-equation upsets the structure of the Z^c-equation. There are, nevertheless, two reasons for considering the Z^c-equation more closely. First, in the period up to 1927, stock-exchange speculation was in fact not very important; secondly, it seems probable that many pre-war cycles can be explained without giving so much weight to the stock exchange. Much the same factors coming into play in the Z^c-equation may have been important in that period, some of them even more important, since the explanation of undamped waves is impossible with the present coefficients.[1]

The economic explanation may be started by repeating the importance of equation (5.10) in the original system, from which the final equation has been derived. This equation states that total profits are the difference between total receipts and total costs of all enterprises. All items in receipts and costs depend, either directly or through a number of channels, on profits (for convenience, corporation profits Z^c have been taken), either at almost the same moment or some time before. The table on page 151 indicates the expression of each of these items in terms of Z^c, together with the result of adding them up or subtracting as may be necessary.[2]

The relative importance of the various components may now easily be seen. In comparing the expressions with the elementary equations, the direct influence of Z^c (which is found there) may be compared with its indirect influence. The most important terms in the table may now be considered separately. A distinction may be made between positive and negative terms, the former tending — if the totals in the columns with

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[1] Except, of course, by the occurrence of fresh shocks (*cf.* section (6.3)).

[2] Exogenous terms are omitted.

Z^c and Z^c_{-1} are larger than 1.44 and 0.27[1] respectively — to reinforce the original deviation in Z, the latter to counteract this tendency.

	Z^c	Z^c_{-1}	Z^c_{-2}	Z^c_{-3}	Z^c_{-4}	No.
Receipts:						
U =	1.420	0.504	0.080	−0.040	0.011	(6.41)
V =	0.467	0.529	0.109	0.046	0.006	(6.42)
Total receipts	1.887	1.033	0.189	0.006	0.017	(6.43)
Costs:						
L_w =	0.529	0.283	0.026	−0.015	0.003	(6.44)
$L_s+L_c+K_I+K_R$ $+N$ =	0.355	0.313	0.262	0.014		(6.45)
Total costs	0.884	0.596	0.288	−0.001	0.003	(6.46)
Difference Z =	1.003	0.437	−0.099	0.007	0.014	(6.47)
Also Z =	1.450	0.260				(5.11)
Therefore 0 =	−0.447	0.177	−0.099	0.007	0.014	(6.48)
App. B, V, 265 * 0 =	−0.445	0.177	−0.098	0.006	0.012	(6.49)

* Small differences between (6.48) and (6.49) are attributable to repeated omissions of small terms.

Big positive influences are those acting through U and V. They express the simple fact that high profits lead to high consumption and high investment. Profits work directly as well as indirectly: on consumption outlay as, *e.g.*, farm prices and farm consumption are high if general incomes are high; on investment outlay as high share prices facilitate high investments and are themselves — through dividends — causally correlated with high profits.

Big negative influences are, apart from the quite natural influence of higher wage totals and other incomes which are paid in times of higher employment, the following.

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[1] These coefficients are those by which Z^c (corporation profits) and Z^c_{-1} must be multiplied in order to yield Z (total profits).

(i) The negative term in U (6.41) is partly due to the influence of commodity stocks. After a year of peak consumption, production falls somewhat more than consumption, as the readjustment of stocks to the lower level of consumption permits of a certain destocking.

(ii) The other part of the negative term in U is due to the influence of speculative gains (capital gains and gains on commodity speculation) on consumption, and thereby on production. These gains are always proportional to some rate of increase, which introduces a positive and a (necessarily somewhat more lagging) negative term, *e.g.*, $Z^c_{-1} - Z^c_{-2}$.

(iii) No negative term will be found in V (6.42); there is, of course, a negative influence of share yield (which represents a type of interest rate) due to its negative sign in (2.4) and the fact that share yield depends positively on dividends, which, in turn, depend on profits; but this negative influence is more than compensated by the direct positive influence of profits. The influences of interest rates in the narrower sense of the term, as well as of prices of investment goods, would also work negatively. These factors were, according to our calculations, almost negligible in the period studied; it is probable that they were stronger in pre-war times, and that they contributed essentially to the formation of cycles in those times.[1]

(iv) The greater incomes paid out (*cf.* (6.44) and (6.45)) at times of higher profits are the consequence not only of higher employment, but also of a higher rate of payment. This may also be a factor tending to reverse the movement of Z^c, especially as there is a lag in the correlation between profits and these rates. This influence is, however, weak, as 0.95 of an increase in wages and salaries is consumed and therefore reflected in U, and since the influence of a lower profit margin on investment activity is, too, not very large (*cf.* relation (2.4)).

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[1] *Cf.* the results given for investment in the United Kingdom in Vol. I.

(6.5) CHARACTER OF MOVEMENTS INTRODUCED BY A STOCK-EXCHANGE BOOM

We have now to consider the second possibility mentioned above, where the rate of increase of our index in stock prices exceeds 20 points per annum. In this case the elimination process may most easily be carried out by the following approximative method. Equation (257), table V,[1] is written in the form

$$n = 3.607Z^c + 1.941Z^c_{-1} - 0.142Z^c_{-2} + 0.012Z^c_{-3} + 2.40\,(\dot{n}_{-\frac{1}{2}} - 20)'' \qquad (6.51)$$

in order clearly to indicate that n is supposed to depend on a preceding rate of increase.[2] It may be combined with equation (6.28) for Z^c (omitting external terms):

$$0.770\,Z^c = 0.179\,Z^c_{-1} + 0.006\,Z^c_{-2} - 0.015\,Z^c_{-3} + 0.007\,Z^c_{-4} + 0.090\,n - 0.049\,n_{-1} + 0.001\,n_{-2} + 0.003\,n_{-3} \qquad (6.52).$$

The character of the movement is now considerably changed; the important fact being that the original form of the Z^c-equation matters much less to the result than the coefficients in the n-equation (6.51). The mathematical solution of equations (6.51/2) shows the movements of the system now to be unstable, *i.e.*, an initial movement in the upward direction will be reinforced in an ever-increasing degree. In order better to understand the character of the changes, we may first study the movements generated by the relation

$$n = 2.40\,(\dot{n}_{-\frac{1}{2}} - 20)'' \qquad (6.51')$$

i.e., relation (6.51) in the assumption of stable profits $Z^c = 0$. The movements may be studied for all values of $\dot{n}$ by assuming, as we did in section (4.8), that the relation between $\dot{n}_{-\frac{1}{2}}$ and n is as follows:

---

[1] External terms omitted.

[2] This, in fact, has been the sense of our equation (4.8). It is only for simplicity that until now $n - n_{-1}$ has been written. Logically, this is less correct, and for that reason it is now dropped.

$$\left.\begin{array}{l} \text{I. For } \dot{n}_{-\frac{1}{2}} < 20: \quad n = 0; \\ \text{II. For } \dot{n}_{-\frac{1}{2}} > 20, \text{ but } < 62: n = 2.40\,(\dot{n}_{-\frac{1}{2}} - 20); \\ \text{III. For } \dot{n}_{-\frac{1}{2}} > 62: \quad n = 100; \end{array}\right\} \quad (6.53).$$

In order to give the problem its simplest shape, we may reduce time units to one-third of their original length, *i.e.*, to four months; $\dot{n}_{-\frac{1}{2}}$ in our previous notation may then be replaced by $3\,(n_{-1} - n_{-2})$, and equation (6.51′) becomes:

$$n = 7.2\,(n_{-1} - n_{-2} - 6.7)'' \qquad (6.51'')$$

which, as before, is only valid for interval II.

Table (6.53) turns into:

$$\left.\begin{array}{l} \text{I. For } n_{-1} - n_{-2} < 6.7: \quad n = 0; \\ \text{II. For } n_{-1} - n_{-2} > 6.7, \text{ but } < 21: \\ \qquad\qquad n = 7.2\,(n_{-1} - n_{-2} - 6.7); \\ \text{III. For } n_{-1} - n_{-2} > 21: \quad n = 100; \end{array}\right\} \quad (6.53').$$

The movements possible under the laws contained in this table are of various types. Starting from an initial level of share prices equal to zero[1] (*i.e.*, some average level), the following possibilities exist.

If no disturbance from outside occurs, the level will remain zero; because $n_1 - n_0$ will be zero, we are in interval I, and $n_2 = 0$; again $n_2 - n_1 = 0$, therefore $n_3 = 0$, etc.

(i) If a small disturbance occurs, viz.,

$$n_1 - n_0 < 6.7,$$

then again $n_2 = 0$; therefore $n_2 - n_1 < 0$, $n_3 = 0$, etc. Share prices will immediately become stable again.

---

[1] It is of no great importance whether this level is indicated by $n = 0$ or by n equal to any other constant. It is essential, however, that the level indicated by 100 is higher.

(ii) A somewhat larger disturbance,

$$n_1 - n_0 > 6.7 \text{ but } < 8.85,$$

has similar consequences. Although n_2 will now be positive, this will not suffice to make its increase over n_1 larger than 6.7; and for $n_2 - n_1 < 6.7$, n_3 will again be zero.[1]

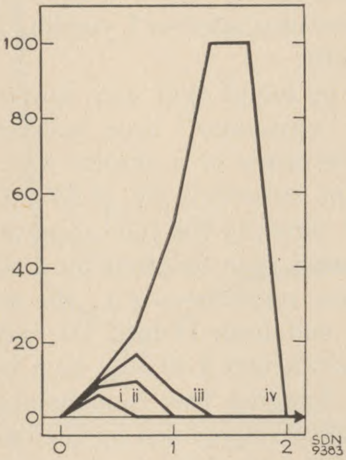
Graph 6.51.

MOVEMENTS OF SHARE PRICES, WITH STABLE PROFITS.

Initial values:

(i) $n_0 = 0$, $n_1 = 6$.
(ii) ,, $n_1 = 8$.
(iii) ,, $n_1 = 9$.
(iv) ,, $n_1 = 10$.

Time in years.



(iii) If $n_1 > 8.85$, n_2 will be at least 15.55, or 6.7 more; therefore n_3 will be positive; but it may be less than n_2; then, and even if it is less than $n_2 + 6.7$, n_4 will again be zero.

---

[1] The limiting value 8.85 for n_1 is found by asking for what value of n_1, $n_2 - n_1$ will be > 6.7; $n_2 - n_1 = 7.2\,(n_1 - n_0 - 6.7) - n_1 = 7.2\,(n_1 - 0 - 6.7) - n_1 = 6.2\,n_1 - 48.2$. In order to make this > 6.7, n_1 has to be larger than $\frac{48.2 + 6.7}{6.2} = 8.85$.

(iv) It can be proved mathematically[1] that if n_1 surpasses 9.5, the movement will be "explosive", *i.e.*, will not return in the way indicated under (i) to (iii). It may be more simple to give an example. Taking $n_1 = 10$, we find:

$$
\begin{aligned}
n_2 &= 7.2\,(10 - 0 - 6.7) = 23.8;\\
n_3 &= 7.2\,(23.8 - 10 - 6.7) = 51.1;\\
n_4 &= 7.2\,(51.1 - 23.8 - 6.7) = 148;\\
n_5 &= 7.2\,(148 - 51.1 - 6.7) = 649.
\end{aligned}
$$

This development, however, stops as soon as the "third interval" is reached, in this case n_4; here the formula $n = 100$ is used instead of the formula $n = 7.2\,(n_{-1} - n_{-2} - 6.7)$. The calculation would continue: $n_4 - n_3 = 100 - 51.1 = 48.9 > 21$, therefore $n_5 = 100$ again; but now $n_5 - n_4 = 0$ and we are brought back at once into interval I, yielding $n_6 = 0$. Graph 6.51 illustrates our results.

It may easily be found that any upward movement, once it passes into the "explosive" type, suddenly falls back upon the zero level in the same or a similar way.

The mechanism described by (6.53′), however simplified, seems to represent correctly the typical movements on the stock exchange; accelerated rises followed by a sudden fall. It may be changed in some respects — *e.g.*, the sudden change from interval I into II and from II into III may be smoothed out somewhat, or the coefficient 2.40 may even be lowered considerably — without changing this fundamental conclusion. The latter must certainly be completed by the remarks that (i) small external shocks may, especially at the beginning, easily interrupt the explosive development, and (ii) the top level of 100 assumed here is of course in a high degree arbitrary.

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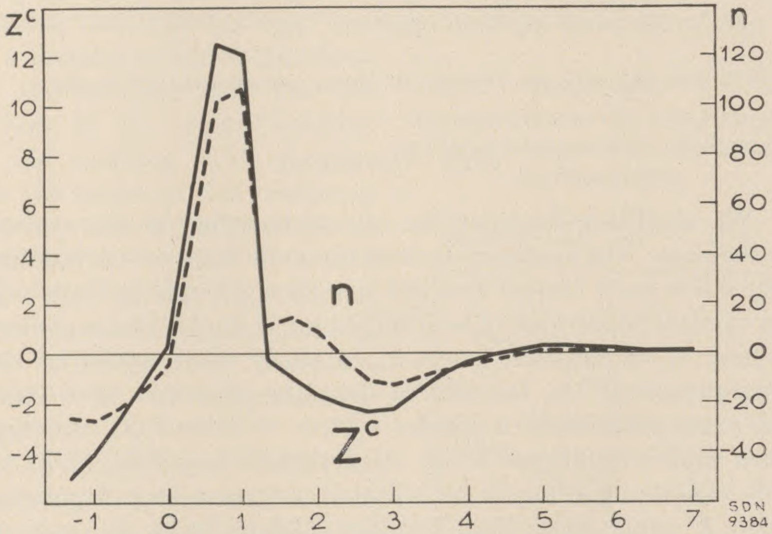
[1] By solving the difference equation $n_t = 7.2\,(n_{t-1} - n_{t-2} - 6.7)$. Introducing as a new variable $n'_t = n_t + 48$, it is homogeneous: $n'_t = 8.7\,n'_{t-1} - 8.7\,n'_{t-2}$, and the roots of the characteristic equation are 6.0 and 1.2. This means that as soon as $\frac{n'_2}{n'_1} > 1.2$ or $n_2 + 48 > 1.2\,(n_1 + 48)$, explosive movements will develop from the start. As $n_2 = 7.2\,(n_1 - 6.7)$, this leads to the condition $n_1 > 9.5$.

But, though the details are incalculable, the main conclusion of the danger of this mechanism to the stability of the system holds. Only if the coefficient 2.40 were less than about 0.33 would the danger of explosive movements be wholly removed.[1]

A combined solution of the equations (6.51) and (6.52) for n and Z^c is very difficult. Only a numerical solution, therefore, has been given in graph 6.52.

Graph 6.52.

Movements of
Corporation Profits (Z^c) and Share Prices (n),
taking Account of the Three Intervals of Share Prices
(Time in years).



It shows one speculative boom, followed by a damped cycle like that of graph 6.32. Evidently it is largely a matter of chance whether or not another speculative boom will occur when the system recovers from the depression ensuing upon the first boom. Quite small shocks could easily lead $\dot{n}$

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[1] Coefficients of this magnitude and lower were found for all other countries investigated in an other publication of the author: "The Dynamics of Share Price Formation". *The Review of Economic Statistics*, November 1939.

into the speculative interval again. After the boom, the asymmetry of the *n*-curve is somewhat less than in the case of graph 6.51, owing to the "support" given to share prices by the high profits still prevailing at the start of the crisis. The period of the Z^c equation is approximately maintained in this combined system.

In the numerical solution reproduced in graph 6.52, a reduction of the time-unit to four months is again needed to bring out adequately the short-term movements connected with a speculative boom. The speculative term in the *n*-equation is again given the form (6.53′), while the other terms in the right-hand members of both equations are adapted to the four-month unit by replacing Z^c_{-1} (lag one year) by

$$\frac{1}{16} Z^c_{-1} + \frac{4}{16} Z^c_{-2} + \frac{6}{16} Z^c_{-3} + \frac{4}{16} Z^c_{-4} + \frac{1}{16} Z^c_{-5}$$

(distributed lag with an average of three four-month units), etc.

(6.6) HOARDING

We shall now consider the rôle of hoarding in the cyclical mechanism. The evidence derived from one instance of hoarding over a few years clearly does not sustain a general conclusion on the cyclical importance of hoarding in the United States economy. It may be of interest, however, to study what would be the consequences if the features of hoarding observed over these few years constituted a regular system of behaviour, recurring when similar conditions recur. As described in section (4.6), we then assume hoarding to be initiated only in a deep depression where Z^c comes more than 7 milliard dollars below its previous peak value. Our form of analysis is not qualified to discover whether or when this situation will occur: the system of equations only describes, as was shown in (6.3), the *propagation* of certain shocks, and it is therefore these shocks that determine, generally, the *amplitude* of the movements, and, in particular, the occurrence of the above situation.

We can only analyse what happens to the cyclical mechanism when the situation is there, *i.e.*, when

$$Z^c_m - Z^c > 7 \qquad (6.61).$$

For that case, the following equation for H was found:

$$H = -0.3\, Z^c \qquad (6.62).$$

By the substitution of this relation for the H-terms in the final equation, we get a new final equation which describes the movements of the system in which hoarding has developed, as long as (6.61) is satisfied:

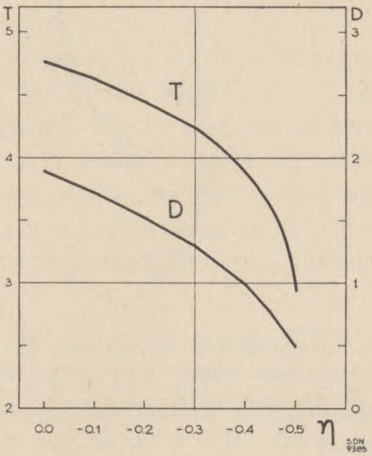
$$Z^c = 0.206\, Z^c_{-1} - 0.508\, Z^c_{-2} + 0.030\, Z^c_{-3} + 0.062\, Z^c_{-4} \qquad (6.63).$$

We may, however, take account of the fact that the coefficient for H in equation (4.91) is not very certain (*cf.* the table on page 113). For this purpose we may study the effects of varying the coefficient of Z^c in equation (6.62) which "explains" H.[1] Graph 6.61 shows the effects on the damping ratio and the period of the cyclical solution of the resulting final equation for the values of this coefficient running from 0 (the general final equation) to -0.5.[2] It is seen that the period is shortened over the whole range. Similarly, the damping ratio decreases, initially at an approximately constant rate.

Graph 6.61.

Damping Ratio (D) and Period (T) as a function of the Intensity of Hoarding.

$H = \eta\, Z^c$,
where $\eta = 0.0, -0.1 \ldots -0.5$.



At about $H = -0.4\, Z^c$, the damping ratio becomes 1; *i.e.*, at values of $H < -0.4\, Z^c$ the cycle becomes anti-damped.

It follows from these figures that the more intensively hoarding occurs, the less the cycle is damped, and it would finally

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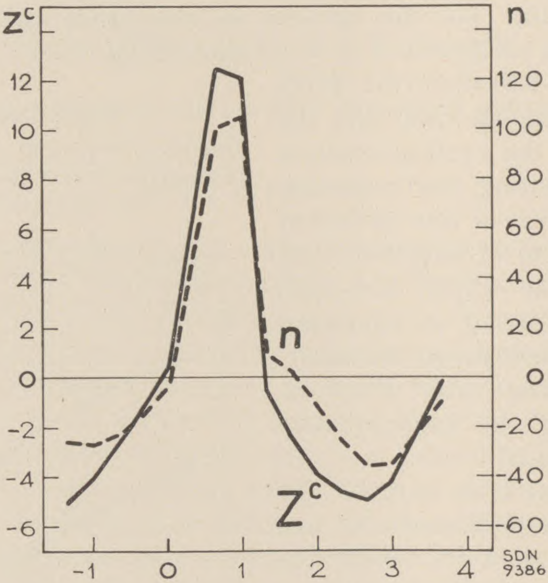
[1] Neglecting the relatively small changes in the other coefficients in these alternative cases.

[2] The calculations are not extended further, since it seems doubtful whether the damping ratio and the period found for $H < -0.5\, Z^c$ have any economic significance.

even become anti-damped when hoarding came to exceed a certain intensity. It should be added that, in any case, the development of Z^c would be described by the final equation (6.63) (or its variant according to other values than — 0.3 in (6.62)) for one or two years at the most: then the depression will be over (*cf.* graph 6.61) and the hoarding condition (6.61) will no longer be fulfilled.

Graph 6.62.

Movements of Corporation Profits (Z^c) and Share Prices (n), under the action of a Speculative Boom, and of Hoarding.



The heavy damping of the system makes it improbable — unless when very large shocks occur — that the movement of Z^c will pass from the normal interval into the hoarding interval. But it is, on the other hand, quite probable that such a heavy fall will occur after a large positive deviation from the average due to a stock-exchange boom.

The possibility of this complication has not been taken into account in section (6.5). We may do this now, and introduce into the numerical solution executed in that section the property that H is replaced by $-0.3\,Z^c$ as soon as, and as long as, $Z^c_m - Z^c > 7$. The result of this changed calculation is shown in graph 6.62.

The period is slightly shortened, to about four years. The downswing is much more rapid than in graph 6.52, and the movement is therefore not unlikely [1] to be swung back into the speculative zone. It does not, however, seem justified to continue the calculation after the depression (the fourth year). Both hoarding and speculation are phenomena in which the psychological factors may change from one cycle to another. Therefore, coefficients determined for past events cannot safely be assumed to hold with about the same values for the future.

In the interpretation of these results, it should be borne in mind that currency-hoarding only has a considerable influence on economic life when, on account of its magnitude in relation to the other items of the balance-sheet of the Federal Reserve Banks, it forces the member banks into debt with the Federal Reserve Banks (*cf.* section (4.4), and especially graph 4.421/2). The continuance of hoarding during the years 1933 to 1937, when it did not prevent the accumulation of excess reserves, cannot, according to the line of our deduction, be held responsible for any considerable depressive influences.[2]

The above treatment of the phenomena of hoarding and stock-exchange speculation may show to what extent the apparatus used in this study is capable, on the basis of an analysis of the elementary relations, of a gradual approach to the complicated and seemingly irregular reality of actual business cycles. In particular, it shows how an illuminating synthesis may be established between these two extreme positions with

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[1] This conclusion must evidently be confined to the indication of certain chances. Whether the actual movement swings back to the speculative interval depends not only on the internal propagation of past shocks according to the final equation, but also on the occurrence and the direction of new shocks.

[2] In this connection, the importance for the United States of the huge gold imports becomes clear (*cf.* section (4.4)). Net imports of gold amounted to nearly \$6 milliard between the depreciation of the dollar and the end of 1937; at that date the excess reserves figured only at \$1 milliard (or, calculated according to the pre-1936 reserve percentages, \$4 milliard).

regard to business cycles: the denial of any regularity in the movements on the one hand, and the assumption of cycles that are strictly regular with regard to period and damping ratio (and perhaps even amplitude) on the other hand. It is especially the rôle attributed to the shocks and the incorporation in the system of a few, but important, non-linear equations which make it possible that one cycle is completely different from another, and that it is yet, in both, one and the same mechanism that links the variables together.

(6.7) BUILDING ACTIVITY AND GENERAL ACTIVITY; THE BUILDING CYCLE; A NEW "MULTIPLIER" CONCEPT

The peculiar rôle played by the housing market in the general economic system has already been mentioned (section (6.3)). Some attention may now be given to the so called "housing cycle". The general business-cycle position, of course, influences house building to some extent, but only in a rather small degree, as a glance at graph 2.5 shows. But the chief factor for house building seems to be the relative shortage or abundance of houses some four years back (of course it is a distributed lag which must be assumed here); and as far as rents are another determinant, these themselves are also influenced to some extent by that shortage or abundance. Putting aside for a moment the smaller factors, we are left with a skeleton of relations:

$$v_B = -0.30\,h_{-4} \tag{6.71}$$

$$\Delta h = h - h_{-1} = 0.92\,v_B \tag{6.72}.$$

It is easily found — as well by simply trying out with arbitrary numbers as by rigorous mathematical treatment — that this set of relations leads to cycles of a period of about sixteen years, which fits fairly well with observations even over a longer period.[1] Although this cycle existed long before

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[1] *Cf.* Roos: *loc. cit.*; Newman: *The Building Industry and Business Cycles*, Chicago, 1935. The method of computation of the period is the one indicated in J. Tinbergen: "Ein Schiffbauzyklus?", *Weltwirtschaftliches Archiv* (34), 1931, II, page 151.

The explanation given by Dr. Roos, *loc. cit.*, page 78, seems to lead to the same conclusion. Dr. Roos explains residential building the war and was remarkably unaffected by the ordinary business cycle, the exceptional reduction of the house-building level during the war accentuated the amplitude of this cycle, and must be held responsible for the exceptional high in 1925 and the exceptional low in 1933. Of course other causes existed as well for the latter low point; but it is remarkable that recovery in private building was so slow in 1934, a fact which fits perfectly well into our present representation. It is very doubtful whether such general statements as "building precedes the business cycle" and "building moves against the cycle" are justified. It all depends on the year observed; in general the direct connection is rather weak.

The fact that v_B shows largely autonomous movements has been one reason for treating it as an external variable. Another is that this procedure enables us to find out the influence of any autonomous increase in building activity on Z^c, and on any other economic variable as well. The most interesting case will be the influence on total volume of production, as this is the well-known problem of the "Multiplier".[1] In order to find the influence of a given increase in building volume v_B on total volume of production $u + v$, this latter variable has been expressed first in terms of Z^c, Z^c_{-1}, Z^c_{-2}, etc., and the external factors:[2]

$$u+v = 1.551\,Z^c + 0.417\,Z^c_{-1} - 0.153\,Z^c_{-2} - 0.097\,Z^c_{-3} + 0.022\,Z^c_{-4} + 1.411\,v_B - 0.257\,(v_B)_{-1} \qquad (6.73).$$

---

activity chiefly by the fluctuations in foreclosure rates two and a half years before. These foreclosure rates themselves seem, however, to be mainly dependent on the relative shortage or abundance of houses a short time before, which influences the profitability of owning houses.

[1] The multiplier concept may be applied as well to employment, income or production, but in any case it should, of course, be a comparison between two phenomena of the same sort. Since v_B has the "dimension" of a volume of production, we have here to apply it to volumes of production.

[2] The other exogenous terms are of no importance in this connection, and are therefore omitted. This is also the case for the terms with h_{-2} and h_{-3}, which relate to influence *via* rents. Since, in this context, v_B represents additional investment activity in general, the repercussions that are particular to investment in dwelling-houses are to be disregarded.

In this expression, Z^c may be replaced by the expression found in the final equation (6.30)[1] and so expressed in terms of Z^c_{-1}, Z^c_{-2}, etc. and v_B[2]. This may be repeated. Owing to the fact that the coefficients of Z^c_{-1}, Z^c_{-2}, etc., in the final equation are smaller than that of Z^c, this procedure leads to ever-decreasing coefficients which after some time become negligible. Thus the procedure finds a natural end, leaving the following formula:

$$u+v = 4.44 v_B + 1.59 (v_B)_{-1} - 0.23 (v_B)_{-2} - 0.57 (v_B)_{-3} - 0.02 (v_B)_{-4} + 0.15 (v_B)_{-5} + 0.04 (v_B)_{-6} - 0.03 (v_B)_{-7} + \dots$$

From this we find that the consequences for total employment of an addition of one unit to building activity in year 1 are, in consecutive years, of the following size:

Year	1	2	3	4	5	6	7	8	9	10	11	12	13
	4.44	1.59	— 0.23	— 0.57	— 0.02	0.15	0.04	— 0.03	— 0.02	0.00	0.01	0.00	0.00

(6.74).

The total of this series, 5.36, is comparable with the old concept of the multiplier.[2] Yet it differs from it in many respects.

(i) The figure represents the ratio between the total effect on production and an initial increase in investment (building activity); this effect consists of u and v, and hence partly of new investment. In the theory of Mr. KEYNES,[3] on the other hand, the multiplier is defined with *total* (and not *initial*) net investment as the denominator.

(ii) The method here employed enables us to take account of a great number of repercussions, *via* many more variables

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[1] After transformation of the term with h_{-1} to v_B and Z^c (equation (2.5); the other terms in this equation may be neglected). The terms with h_{-2} and h_{-3} are disregarded (*cf.* page 163, note 2). The omittance of the q_B-term in (2.5) may be interpreted that it is cancelled out by the q_B-term in (1.16) (*cf.* section (3.6)); it follows, then, that the results found refer not to the execution of a certain *volume* of "public works", but to the spending of a certain *sum*.

[2] A calculation along the lines indicated by Mr. KEYNES would give about 6 (Method of calculation, *cf.* POLAK, *loc. cit.*, page 10).

[3] *The General Theory of Employment, Interest and Money*, London, 1936.

than can be allowed for by a procedure such as that first developed by Mr. R. F. Kahn.[1]

(iii) Previous calculations found the multiplier as the sum of the terms of a geometric series with a positive ratio; no negative terms occurred in this sum. The series (6.74), on the other hand, shows an alternation of positive and negative terms. This is due to the fact that it may be considered as the sum of:

(*a*) $1.411 v_B - 0.257 (v_B)_{-1}$ in (6.73);

(*b*) Four superposed damped cycles of the period and damping of the final equation (*cf.* graph 6.32), weighted with coefficients 1.551, 0.417, etc., and with a phase-difference of one year between any two cycles.

It will be clear, then, not only that part (*b*) contains certain negative figures, but also that its net sum over an infinitely long period of time is only positive when the cycle is damped and the sum of the weights is > 0. If the cycle had been undamped, the sum of (*b*) over a certain number of years would be alternatively increasing and decreasing, passing through 0 after any 4.8 years.

(iv) It is clearly shown in this approach that the value found for the multiplier is subject to a certain restriction. We have used the final equation for Z^c based on the "normal" interval in the *n*-equation. Since the multiplier concept has been generally used for depression periods, this is justifiable; but it means that, if by the autonomous addition to building volume a boom develops, the formula is no longer valid. It would lead too far to go into the problems involved.[2]

Another restriction to be made is that, if the "public works" represented here by v_B were to consist in the building of residences, we should have to take account of the fact that, according to equation (2.5), to-day's building "spoils the market" for the future. This would give additional negative terms in (6.74) after the third year.

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[1] "The Relation of Home Investment to Unemployment", *Economic Journal*, XLI (1931), page 173.

[2] *Cf.* also J. Tinbergen: "Über die Sekundärwirkungen zusätzlicher Investitionen", *Weltwirtschaftliches Archiv* (45), 1937, I, page 39.

(6.8) THE FORMAL CHARACTERISTICS OF BUSINESS-CYCLE POLICY

In the preceding sections, certain characteristics, in different circumstances, of the business cycle of the United States were discussed. We may now turn to the analysis of the effects on the character of this cyclic movement of measures of policy and of some much-discussed changes in the economic structure.

For a good understanding of the problems to be treated here, some methodological problems must again be touched upon.

(i) Measures of policy may, in accordance with the analysis in section (6.3), be grouped as:

(*a*) Changes in the coefficients or lags, or both, *i.e.*, changes in the economic structure of society — *e.g.*, price-stabilising measures;

(*b*) Shocks — *e.g.*, the Veterans' bonus payment in the United States in 1936;

(*c*) Changes in the average level of some variable — *e.g.*, minimum wage legislation. The effects of this last type of change are not taken into account in this study, which is concerned specially with the problem of business cycles.

The apparatus for analysing the effects of shocks has been discussed in section (6.7), where a more refined and qualified multiplier-concept has been elaborated. Nothing more need be said about these here, especially since a policy that takes the form of irregular disturbances will in general not be considered desirable, since it would increase uncertainty instead of decreasing it.

In this section we shall therefore restrict the analysis to the effects of changes in coefficients and lags.

(ii) Any measure, therefore, which is not defined in sufficient detail to be translated into a definite change in some definite coefficient cannot be discussed at all. As an example, take a proposal to make consumption less dependent on the business cycle; *e.g.*, by higher taxes during the boom, lower taxes during depression, and more stabilised ordinary expenditure by the State. Since we have seen (table, section (6.4)) that consumption outlay depends on Z^c, Z^c_{-1}, Z^c_{-2}, Z^c_{-3} and Z^c_{-4}, five coefficients might be affected by this proposal. It makes some difference which of these coefficients is changed most; measures of a given intensity but with different lags may have quite different results.[1]

(iii) In the study of the effects of a change in one coefficient, it must not be forgotten that we have not analysed why the coefficients are as large as we have found them to be; we have simply determined their magnitude by the multiple correlation technique. Hence it is not sure beforehand that a change in one coefficient will leave all other coefficients as they are: some coefficients may be linked to one another by relations into which we did not enquire: *e.g.*, a stabilisation of dividends may influence the way in which shareholders appreciate a certain value of d — *i.e.*, the coefficient ν_1 in equation (4.81). The so-called "variation problem" to be attacked now must therefore be handled with care in this respect.

(iv) The result of these variation calculations has various aspects. Any cyclic movement is characterised by period, damping degree, phase and amplitude, and each of these four may be changed. In some discussions too little care seems to have been taken to distinguish between these types of change. Such is the case in the well-known question whether the boom can be continued (once a peak level has been reached) by increased consumption or increased saving. Such a discussion generally bears upon an incidental lengthening of the boom; and such a lengthening may be the consequence of a change in phase as well as a change in period, damping degree or amplitude. The distinction between these has seldom been discussed. Nevertheless, it seems necessary to do so since, *e.g.*, changes in phase are far less important than changes in damping degree and amplitude. It is especially these latter which a systematic stabilisation tries to change. In the following pages only the

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[1] The more important distinction, between changing the average level of consumption and changing the fluctuations in consumption, is often forgotten in more popular discussions. Here, of course, we are speaking only of the latter.

very first beginnings of the questions involved are indicated. Much more space would be required — and in fact it would involve a new subject — if these questions were to be dealt with completely.

We shall treat the variation problem in two ways. In the first place it will be assumed that for a certain variable *all* fluctuations are excluded, *i.e.*, that all coefficients in the equation "explaining" this variable are taken as zero. Secondly, the effects of relatively small changes in a number of important coefficients and lags will be taken up one by one.

It should be noted that these calculations not only give an estimate of what would be the characteristics of the American business cycle when certain changes in the equations are brought about (the problem of policy), but they also tell us to what extent our idea of the present mechanism may be falsified by the fact that some of our equations afford an inaccurate picture of reality (margins of error). The same effects on our final equation are caused by a certain coefficient's *being* in reality 5 % lower than it *seems* to be, or by its *becoming* 5 % lower than it *is*.

For reasons of exposition we start again with what has been called the "normal interval", *i.e.*, we assume the absence of a stock-exchange boom and of exceptional hoarding. Given the importance of speculation or hoarding for economic life as a whole, this point must not be neglected.

(6.9) THE EFFECT OF SOME MEASURES OF BUSINESS-CYCLE POLICY OR CHANGES IN THE ECONOMIC STRUCTURE ON THE CHARACTER OF THE CYCLIC MOVEMENT

I. *All Coefficients in one Equation taken as Zero.*

First, let us consider the consequences of a complete stabilisation[1] of investment activity, *e.g.*, by compensatory public investment.

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[1] This term must be understood not to mean that investment outlay would always have the same constant value, but that it would have some smoothly increasing ("trend") value. For some of the implications involved *cf.* J. TINBERGEN: *Fondements mathématiques de la stabilisation des affaires* (Hermann, Paris, 1938).

Such a stabilisation of investment outlay assumes that all coefficients in the expression determining *v* are zero. As, in formula 216 (Appendix B, table V), *v* is still included explicitly, the corresponding final equation is easily calculated.

This final equation would run (apart from external terms):

$$0 = -0.76Z^c - 0.26Z^c_{-1} - 0.15Z^c_{-2} - 0.21Z^c_{-3} \text{ or}$$
$$Z^c = -0.34Z^c_{-1} - 0.20Z^c_{-2} - 0.03Z^c_{-3}, \qquad (6.91)$$

and the corresponding cycles would be substantially more damped than the original cycles, whereas the period would be shorter (*cf.* table 6.91).

Table 6.91.

The Effect of Some Measures of Policy on the Damping Ratio and Period of the Cycle.

Case	Period in years	Damping ratio
No policy	4.8	1.89
Stable investment outlay . .	3.8	2.41
Stable consumption outlay .	3.2	2.62
Rigid wages	4.7	1.85
Rigid prices	4.3	1.71

Second, let us consider a stabilisation of consumption. This might be the consequence of a change in human attitude, leading to an increased rate of savings in boom periods and a decreased rate of savings in depressions. It might also be the consequence of Government policy, as has been observed already — *e.g.*, by "compensating taxes" and a stabilisation of ordinary State expenditure. Finally, it might also partially be obtained by a less unequal distribution of incomes. The consequences can be found by much the same method as used in the case of investment stabilisation.

The final equation therefore becomes:

$$0 = -1.41Z^c - 0.17Z^c_{-1} - 0.16Z^c_{-2} - 0.03Z^c_{-3} \text{ or}$$
$$Z^c = -0.12Z^c_{-1} - 0.12Z^c_{-2} + 0.02Z^c_{-3}, \qquad (6.92)$$

and the corresponding movements are even more damped than those obtained in the previous case.

In the third place, the consequences of changes in the flexibility of wages, and in the fourth place those of changes in the flexibility of prices, will be considered. Both problems have attracted a good deal of attention in economic literature. It has been held that the rigidity of wages and prices is responsible for the increased amplitude of business cycles in recent times. On the other hand, stabilisation of prices has been advocated as a means of stabilising general activity. The treatment of these problems seemed easiest when (i) wage rates l_w and (ii) all prices (p, p', p^f, q, q_B and m_r) were assumed to be absolutely rigid, and the final equations were recalculated on this basis.

They run:

$$\text{Wage rigidity: } Z^c = 0.14Z^c_{-1} - 0.25Z^c_{-2} - 0.04Z^c_{-3} \qquad (6.93)$$

$$\text{Price rigidity: } Z^c = - 0.01Z^c_{-1} - 0.35Z^c_{-2} - 0.05Z^c_{-3} \qquad (6.94)$$

as against the normal case:

$$Z^c = 0.40Z^c_{-1} - 0.22Z^c_{-2} + 0.01Z^c_{-3} + 0.03Z^c_{-4} \qquad (6.36).$$

The case of wage rigidity hardly differs from the normal case with respect to damping ratio and period. This means that, at least in the United States, wage rigidity is not so detrimental to a stabilisation of cyclic movements as has sometimes been believed.

According to these calculations, price rigidity or price stabilisation[1] would have had a somewhat anti-damping effect. This result should, however, be accepted with some caution,

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[1] These two words represent, of course, two very different types of policy. Price rigidity is commonly understood to be caused by monopolistic tendencies, and to be a form of policy pursued by private concerns or groups. Price stabilisation covers a far wider field; it may be used for the same private ends, but also for governmental action in which varying instruments are brought into play in order to attain a stable price level. Of this latter class, only such measures are meant here as act directly on prices themselves; *e.g.*, a policy of holding stocks of raw materials, or using raw materials as cover for note circulation,

since the coefficients for the price-variables are rather uncertain in some of the equations (*e.g.*, (2.1) and (2.4)).

Finally, it must not be overlooked that the foregoing calculations are all only valid for a period with no stock-exchange boom. Although there is no doubt that a stabilised economy would offer fewer opportunities for the development of a stock-exchange boom, it is still possible that even with a perfectly stabilised endogenous development some incidental cause may lead to a speculative boom, as described in section (6.5). Unless, therefore, the price-formation of shares is changed considerably, this part of the mechanism will continue to be a threat to stability in economic life. It follows from the above that those proposals for stabilising the so-called "general price level", in which share prices are also included, may be of still more importance than those that aim only at stabilising the prices of goods and services — provided that the method for obtaining stabilisation is indicated clearly.

II. *Changes in Coefficients.*

The most important coefficients, the effects of changes in which we wish to study particularly, occur in the equations determining

(2.1) Consumption;
(2.4) Investment;
(2.6) Stocks;
(3.5) Prices of capital goods;
(4.8) Share prices;
(5.1) Dividends;
(5.2) Entrepreneurial withdrawals;
(5.3) Capital gains;

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or, finally, regulating prices by decree or by subsidies. Such more indirect measures of price stabilisation as act through the volume of credit, for example, are not included. They could better be designated by terms indicating what instrument is used (*e.g.*, credit rationing, discount-rate policy, etc.); and in order to discover their consequences other calculations than those made here are necessary.

which largely coincide with the equations indicated above as "strategic" equations.

We may write these equations as follows (dots indicating the terms in each equation in which no changes are made):[1]

$$
\begin{aligned}
&(\mathrm{V}2.1) & \mathrm{U}' &= (0.95 + \upsilon^{\mathrm{L}})(\mathrm{L}_w + \mathrm{L}_s) + (0.77 + \upsilon^{\mathrm{E}})\,\mathrm{E} + \mathrm{E}'_{\mathrm{F}} \\
& & &\quad + 0.28\mathrm{G}\,\text{[2]} + (0.049 + \upsilon^{\Delta})\,\Delta p^f + (0.03 + \upsilon^p)\,p \\
&(\mathrm{V}2.4) & v' &= (0.33 + \varphi^z)\,\mathrm{Z}^c + (0.33 + \varphi_1^z)\,\mathrm{Z}^c_{-1} - (0.47 + \varphi^m) \\
& & &\quad [m_{\mathrm{L}s} + (m_{\mathrm{L}s})_{-1}] - \dots \\
&(\mathrm{V}2.6) & w &= (0.105 + {}_{F})\,u' + (0.047 + {}_{F_1})\,u'_{-1} \\
&(\mathrm{V}3.5) & q &= 0.35\,l + (1.29 + \chi)\,v'_{-0.46} \\
&(\mathrm{V}4.8) & n &= (20.6) + \nu)\,d - \dots \\
&(\mathrm{V}5.1) & \mathrm{D} &= (0.151 + \delta)\,\mathrm{Z}^c + (0.083 + \delta_1)\,\mathrm{Z}^c_{-1} + \dots \\
&(\mathrm{V}5.2) & \mathrm{E}_{\mathrm{E}} &= (0.110 + \varepsilon)\,\mathrm{Z}^c + (0.066 + \varepsilon_1)\,\mathrm{Z}^c_{-1} + \dots \\
&(\mathrm{V}5.3) & \mathrm{G} &= (0.200 + \gamma)\,\dot{n}_{-\frac{1}{2}}
\end{aligned}
$$

The Greek letters υ^{L}, υ^{E} . . ., etc., represent relatively small variations in the coefficients together with which they occur. The elimination process may now be repeated with the changed coefficients, *i.e.*, carrying on the variations to the coefficients as algebraic symbols[3] throughout the process. In this way, a final equation[4] is obtained, which contains terms with $\upsilon^{\mathrm{L}}\mathrm{Z}^c$, $\upsilon^{\mathrm{L}}\mathrm{Z}^c_{-1}$, . . . $\upsilon^{\mathrm{E}}\mathrm{Z}^c$, $\upsilon^{\mathrm{E}}\mathrm{Z}^c_{-1}$ . . . etc.

From this equation we may study alternatively the dependence of damping ratio and period of the resulting cyclical

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[1] The V before the numbers indicates that the equations are obtained by *variation* of (2.1), etc.

[2] It is not necessary to treat separately the coefficient for G in this equation, since the effects of a certain change in it are equivalent to those of a change of the same relative magnitude in the coefficient γ (equation (V5.3)).

[3] As the effect of small variations only is to be studied, products or second and higher powers of variations in coefficients have been neglected.

[4] The laborious process of differentiating each intermediate coefficient in the whole elimination process with respect to these sixteen variables results in a mass of figures which it would require too much space to publish.

movement on small variations in each of the coefficients chosen (by equating to zero all but one of the variations). The response to a 10% change in each coefficient is given in the following table:

Table 6.92.

EFFECT ON DAMPING RATIO AND PERIOD OF A 10% INCREASE IN CERTAIN COEFFICIENTS.

Equation	Determining	Coefficient of:	Magnitude in equation	Change in:	
				Damping ratio	Period (years)
(2.1)	U'	(L_w+L_s)	0.95	— 0.147	0.56
		E	0.77	— 0.033	0.18
		G	0.28	— 0.134	— 0.07
		Δp^f	0.049	— 0.039	0.09
		p	0.03*	0.072*	0.27*
(2.4)	v'	Z^c	0.33	— 0.018	0.02
		Z^c_{-1}	0.33	0.011	0.07
		Lag of Z^c	*0.50*	*0.029*	*0.05*
		$m_{Ls}+(m_{Ls})_{-1}$	— 0.47	— 0.0004	0.02
(2.6)	w	u'	0.105	— 0.051	— 0.15
		u'_{-1}	0.047	— 0.069	0.05
		Lag of u'	*0.31*	*— 0.046*	*0.12*
(3.5)	q	$v'_{-0.46}$	1.29	— 0.010	— 0.01
(4.8)	n	d	2.06	— 0.119	0.02
(5.1)	D	Z^c	0.151	— 0.082	— 0.11
		Z^c_{-1}	0.083	— 0.082	0.13
		Lag of Z^c	*0.35*	*— 0.037*	*0.19*
(5.2)	E_E	Z^c	0.110	— 0.019	0.01
		Z^c_{-1}	0.066	0.006	0.03
		Lag of Z^c	*0.38*	*0.017*	*0.02*
(5.3)	G	$\dot{n}_{-\frac{1}{2}}$	0.200	— 0.134	— 0.07

* Since this coefficient represents a minimum, rather than an average value, the variation has been calculated for a 100% increase.

For the treatment of this variation problem, the variant (5.7′) has been chosen for the L_s-equation (*cf.* pages 120 and 146). This choice was due to the appearance of the somewhat improbable values (due to the term with Z^c_{-2} in (5.7)), in table 6.93 which are given below as they would have been with equation (5.7) instead of (5.7′), as far as the difference is considerable ($>$ 10% of the values of (6.92)).

Again, values corresponding to any intermediate equation for L_s, between (5.7) and (5.7′), may be approximately calculated by interpolation between the figures of tables 6.92 and 6.93.

It is, however, believed, in particular on the ground of the sign attaching to the influence of υ^L on the damping ratio in table 6.93, that the figures of table 6.92 are nearer to reality.

Table 6.93.

Equation	Coefficients of:	Change in:	
		Damping ratio	Period
(2.1)	$L_w + L_s$	0.051	0.49
	p	0.105	0.11
	Δpf	— 0.020	0.08
(2.6)	u'	— 0.063	— 0.07
	u'_{-1}	— 0.046	0.08
	Lag in u′	*— 0.018*	*0.11*

Returning to table 6.92, the general impression is that an increase in one of the coefficients in nearly all cases causes (i) a decrease in the damping ratio, and (ii) an increase in the period.

The table enables us to see what coefficients are of the greatest importance for the characteristics of the cyclical mechanism.

(2.1) Most coefficients in the consumption equation prove to be important; especially those of the series $L_w + L_s$ (with the largest standard deviation) and of G. It should be emphasised that the latter is one of the few coefficients, an increase in which causes a decrease in the period. The figures confirm the theory that lower marginal propensities to consume are an important objective for a stabilisation policy.

(2.2) Changes of the coefficients of Z^c and Z^c_{-1} in the same direction have opposite effects on the damping ratio. Hence the relative importance of a change in the lag. An increase in the lag may be represented by a decrease of the coefficient of Z^c, combined with an increase of the same magnitude in the coefficient of Z^c_{-1}; these two changes affect the damping ratio in the same direction, so as to make a larger time-lag in the investment decisions of entrepreneurs conducive to a reduction in cyclical fluctuations.

The results found with respect to this equation are, again, in harmony with what we found above — viz., that an increase in the fluctuations in investment activity intensifies the cycle.[1] The influence of changes in the m_{Ls}- coefficient is very small.

(2.6) The rôle of stocks of consumption goods proves to be rather important; if stocks were constant, and, hence, the coefficients in equation (2.6) were both zero, the damping ratio would be larger by $10 \times (0.051 + 0.069) = 1.20$.

(3.5) The coefficient of 1.29 in this equation is an inverted measure of the elasticity of supply. By varying this coefficient, we may find out how the cycle is changed by a change in the elasticity of supply of capital goods — *e.g.*, as a consequence of a change in the organisation of the market — or when bottlenecks occur.

It was estimated in section (3.5) that, owing to the first of these two events, the coefficient for v' in the "explanation" of q had been about 3×1.29 in the years 1919, 1921 and 1922, and that it had been considerably larger in the bottle-neck years 1920 and 1923. If we take as a rough figure a 500% increase in the coefficient as representative of this latter situation, we find :

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[1] The figures show that this statement is only true for investments that are less than about eight months lagged behind Z^c ($0.18 \div 0.29 \times 12$ months). This would imply that public works, even if executed without a purposive policy of timing, would in fact have a certain damping effect, provided that they lagged sufficiently behind profits in private enterprise (which will often be the case). It will, of course, be clear that this effect may be greatly increased by well-balanced timing of public works.

In the case of	Coefficient increased by	Damping ratio	Period
Less monopolistic organisation .	200%	1.69	4.6
Bottle-necks	500%	1.39	4.3

It will be seen that a decrease in the elasticity of supply of capital goods diminishes the damping of the cycle; in the case of very serious bottle-necks, the cycle may even become anti-damped.

(4.8) Given the large rôle played by the share price in the system of equations, it is no surprise to find that a 10% increase in the coefficient of its most important determinant, *d*, has an appreciable negative influence on the damping of the system. Since, however, this coefficient is known with a considerable degree of precision (*cf.* table on page 113), there is not much danger of a serious error in the damping ratio on its account.

(5.1) The influence found for a 10% change in the dependence of dividends on profits (current and for the preceding year) is evidently still greater than that of an equally large variation in the *d*-coefficient in (4.8), where the effects via consumption are not included. The figures found seem especially interesting with regard to measures of policy: if the distribution of dividends could be made to be more stable, say by 25%, the damping ratio of the cycle would increase by $2.5 \times (0.082 + 0.082)$ or 0.410. The period proves to be very sensitive to changes in the lag of dividends behind profits. It may be deduced that the quick reaction of dividends to changes in profits (with an average lag of 0.35 of a year) might well be one of the main factors making for the difference in period between the European and the American cycle. If the lag of dividends behind profits were twice as large — *i.e.*, eight to nine months, or about at the magnitude it probably has in most countries in Europe — the period would be about two years longer.[1]

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[1] To check whether the use of the figures in table 6.92 is, in this case, legitimate for so large a change in the coefficients, the elimination process has been repeated with an equation for D with a lag of nine months: $D = 0.059\, Z^c + 0.175\, Z^c_{-1}$. This calculation confirmed the prolongation of the period.

(5.2) With regard to this equation, which "explains" E_E, the influence of the lag on the characteristics of the cycle is of particular importance. According to the coefficients in equations (5.1), (5.2), (5.6), $3(E_E - E'_F - E''_F)$ is about equal to $E_E - E'_F - E''_F + D + L_c$, the constituents of which form the most fluctuating items in E (equation (1.3)). Hence an increase of one month in the lag of entrepreneurial withdrawals behind profits is, in its effects on damping and period, equivalent to an increase in the lag of consumption due to business income $E_E - E'_F - E''_F + D + L_c$ behind these incomes by about one third of a month. In this way, the calculated effects of the variation in the lag in equation (5.2) may serve to determine whether a possible small lag of consumption of non-workers behind their incomes, of which we found no evidence in section (2.1), might have had an appreciable effect on the damping ratio and the period. It appears that, if the incomes $E_E - E'_F - E''_F + D + L_c$ entered in the consumption equation with a lag of three months, the damping ratio would be 0.35 higher and the period half a year longer.

(5.3) Finally, the coefficient for $\dot{n}_{-\frac{1}{2}}$ in the equation "explaining" G proves to be of very great importance. If the speculative income arising from a given rise in share prices became twice as large — or if consumption reacted with double intensity to speculative gains[1] — the cycle would become heavily anti-damped. It follows, as it does from the features of the "speculative interval" described above (6.5), that, in the period and country under review, a policy directed to diminish speculation would have a stabilising effect.

The discussion of the effects of the variation of individual coefficients on the damping ratio and the period of the cycle may also serve to determine the uncertainty of the figures found for these magnitudes (1.89 and 4.8 years). Here a possible offsetting of the effects of various coefficients in one equation must be taken into account. Let us consider the consumption-

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[1] This comes to the same, since the only place in the equation system where G occurs is (2.1). Hence the variations with respect to (5.3) could directly be applied to the G-coefficient in (2.1).

equation (2.1). From a comparison[1] between cases 1*a* and 1*b*, and 2*a* and 2*b*, we find that a decrease in the coefficient of $L_w + L_s$ is accompanied by twice as large an increase in the coefficient of E. Accordingly, when the coefficient of $L_w + L_s$ changes by 10% (the case treated in table 6.92), that for E must change in the opposite direction, and by 25%.[2] Similarly, a comparison of cases 4*b* and 1*b*, 6*a* and 2*a* and 6*b* and 2*b* shows that, at a given coefficient for $L_w + L_s$, an increase in the coefficient for *p* is accompanied by 2.5 times as large a fall in the coefficient for E; hence a 10% decrease in the latter is to be compared with a 100% increase in the former. The effects of these *combined variations* are shown below:

Coefficient of:	Change	Effect on:	
		Damping ratio	Period
$L_w + L_s$	— 10%	0.147	— 0.56
E	+ 25%	— 0.083	0.45
Combined effect		0.064	— 0.11
E	— 10%	0.033	— 0.18
p	+ 100%	0.072	0.27
Combined effect		0.105	0.09

It will be seen that, whereas the effects of compensatory changes in the coefficients of $L_w + L_s$ and E cancel out to a great extent, this is not the case for the damping ratio when the coefficients of E and *p* (or of $L_w + L^s$ and *p*) are varied in this way.

To give an idea of the extent to which uncertainty with regard to the final equation is due to equation (2.1), the damping ratio and the period of the latter have been calculated for what would seem to be two extreme cases. The first (I) is case 2*a*,[3] the other (II) is derived from a comparison of cases 6*b* and 2*b*, but with a fixed coefficient of 0.30 for *p* (which means a price elasticity of demand for consumers' goods of $\frac{1}{2}$)[4]. The

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[1] *Cf.* page 37. The figures in other cases are disturbed by the inclusion of other series, which lead to multicollinearity.

[2] $2 \times 10 \times 0.95 \div 0.77$. Changes in the other coefficients are for a moment left out of account.

[3] *Cf.* page 37.

[4] *Cf.* equation (1.10).

coefficients for the various variables and the corresponding damping ratio and period run as follows:

	Coefficient for:					Damping ratio	Period (years)
	$L_w + L_s$	E	G	$D_p f$	p		
Case chosen . .	0.95	0.77	0.28	0.049	0.03	1.89	4.8
Extreme I . . .	1.00	0.75	0.27	0.046	—	1.82	4.7
Extreme II . . .	0.80	0.41	0.29	0.061	0.30	2.78	5.7

It follows that the errors which may be present in the consumption equation chosen would tend to cause too low a damping ratio and too short a period, rather than the opposite.

To be able to estimate the total probable error in the damping ratio and the period, we should know:

(i) The probable error in all elementary coefficients;

(ii) The degree of (positive or negative) interdependence between the probable errors of the coefficients within each equation;

(iii) The derivative of the damping ratio and the period with respect to all elementary coefficients.

Each of these three requirements is only partly fulfilled in the present investigation. Hence we cannot estimate the exact amount of the probable error in the final results. To arrive, however, at a figure from which an impression of the order of magnitude of these errors may be obtained, the error in the damping ratio and the period is calculated on these assumptions:

(i) The seventeen coefficients mentioned in table 6.92 (page 173) have each a standard error of $\pm$ 10% of the value of the coefficient;

(ii) These standard errors are independent;

(iii) The other coefficients are free of error.

On these assumptions we find:

for the damping ratio, 1.89 $\pm$ 0.32;
for the period, 4.8 $\pm$ 0.7.

CHAPTER VII

CRITICAL CONCLUSIONS ON SOME BUSINESS-CYCLE THEORIES

(7.1) INTRODUCTION; RESERVATIONS TO BE MADE

The foregoing analysis of the business-cycle mechanism makes it possible to draw a number of conclusions concerning the validity of some of the theories of the business cycle. These conclusions are subject to numerous limitations which may be shortly summarised here; further details will be found in the sections dealing with the separate relations.

(i) The period and country considered are in many respects special. It has even been said that no business cycles have occurred in the post-war period (CASSEL,[1] HAWTREY).[2] Without going so far, it may be stated that the analysis showed many abnormal features. Up to 1927, the development was fairly stable[3]; the occurrence, however, of condition favouring stock-exchange speculation — in our terms: $n - n_{-1} > 20$ — brought about a fundamental change: both the boom up to 1929 and the following depression showed rather an anti-damped character. Another exceptional feature was the absence of any considerable rise in prices in 1929.

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[1] *The Theory of Social Economy*, Vol. II, page 538: "The economic development of post-war times has been so strikingly dominated by great monetary disturbances that trade cycles of the earlier kind are no longer applicable."

[2] *Cf.* HABERLER, *Prosperity and Depression*, revised and enlarged edition, 1939, page 14.

[3] " ... a brief examination of the period 1922-1929 shows shat the cyclical fluctuations have been notably moderate." *Recent Economic Changes*, Report of the Committee on Recent Economic Changes (New York, 1929) Vol. I, page 12.

(ii) Some important statistics used are admittedly incomplete.

(iii) Only slow changes in the coefficients have generally been assumed to take place.

(iv) Important explanatory factors for some of the variables included may have been omitted.

(v) The determination of some of the regression coefficients is interfered with by " multicollinearity ". This does not necessarily invalidate the results.

An example of a certain compensation of errors, in connection with the consumption equation, has been elaborated in the previous chapter. There are a number of similar cases in which the uncertainty of coefficients due to multicollinearity is not important for the final results. But if, for example, the series used for consumption had to be replaced by another estimate showing different fluctuations, then a revision of some results might be necessary.

(7.2) Difficulties in the Classification of Theories

Let us begin with some remarks on the classification of theories. A first distinction may be made between exogenous and endogenous theories. By an exogenous theory we mean a theory explaining cyclic movements by cycles in one or several of the "data " — *i.e.*, of the non-economic phenomena (such as crops or psychology).

Endogenous theories, on the other hand, explain the cycles without the help of cycles in data. In the Introduction and in Chapter VI, we saw that the following conditions must be fulfilled if an endogenous cyclic movement is to develop:

(i) At least one of the relations must be dynamic — *i.e.*, must contain variables relating to different time-points (as special cases, differentials and cumulants of variables may be mentioned);

(ii) There must be an initial disturbance of the system;

(iii) The final equation must fulfil certain conditions; otherwise, either a cumulative or a one-sided damped movement only may develop. These conditions have been enumerated in detail for a second-degree characteristic equation; it would take us too far afield to give them for more complicated cases.

In the light of this knowledge, let us now consider how a logical classification of the various possible endogenous theories can be made. It is obvious that dynamic features appear in one or other of the various relations. Independently of this, the initial disturbances may occur in different parts of the system. In principle, therefore, the dynamic features in a system of variables may be present in equation 1, equation 2, equation 3, and so on, or in any pair of such equations, or in any three, etc. The same is true of the disturbances. A classification could therefore be made either according to the localisation of the dynamic features or according to the localisation of the disturbances. This, however, would lead to a very large number of possible theories, even if we considered only the most important relations, leaving out, for example, the definitional equations and those "explaining" the variables of minor importance.

Turning to the theories actually put forward by various authors, it appears that many of them are not complete in the sense of dealing with all coefficients and lags necessary to establish the equations. Some emphasise one dynamic feature (*e.g.*, Aftalion's theory; the acceleration principle); others, certain disturbances (*e.g.*, the agricultural theories). Others again do not explicitly state dynamic features, although they contain them implicitly. The over-investment and under-consumption theories are of this type. Taking the very simple case of over-production in a certain part of the system, we find on closer examination that most explanations imply either that unexpected additions to production (*i.e.*, disturbances) occur, or that production takes some time (*i.e.*, a dynamic feature), or, thirdly, that it is influenced by the rate of increase in some variable (another dynamic feature).

Finally, some of the general theories draw special attention to various sorts of "bottlenecks". The latter are not, as such, dynamic relations, but, in our language, curvilinear relations, in which each coefficient takes different values at different distances from equilibrium. For systems containing such relations, the likelihood that condition (iii) above is fulfilled is greater at least for some values of the variables. Thus, whereas curvilinear relations are not sufficient to bring about cyclic movements if dynamic features are not present in the system, they may cause a non-cyclic (cumulative) movement to become cyclic at a definite distance from equilibrium. This, by the way, is the *raison d'être* of Professor HABERLER's subdivision, in his own theory, of the cycle into four parts: two cumulative processes and two turning-points.

In conclusion, it may be stated that the points stressed as essential by various authors come under one or other of (i), (ii), or (iii) above, and this makes it difficult to give a logical classification of their theories. That is probably the reason why, in the usual classification — also followed by Professor HABERLER in his book — it is not always clear which of the above-mentioned aspects has been chosen as the principle of classification.

With these considerations in mind, let us see what is the place in our system of the relations stressed by certain prominent theories, and their relative importance for the cyclical mechanism. We shall adopt the same classification as Professor HABERLER: the rôle of monetary factors may therefore be considered first.

(7.3) THE RÔLE OF MONETARY FACTORS

Professor HABERLER says:

> "Money and credit occupy such a central position in our economic system that it is almost certain that they play an important rôle in bringing about the business cycle, either as an impelling force or as a conditioning factor." [1]

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[1] HABERLER, *loc. cit.*, page 14. The reference to the "impelling force" applies more especially to the supply side of the market, and particularly to a deliberate pressure on the interest rate by the banking system.

Our study leads to the following conclusions in this respect:

(*a*) The influence of interest rates, in the restricted sense of discount rates and other short-term rates, on goods is found to have been very small (equation (2.4)).

(*b*) The influence of long-term interest rates on investment activity in durable goods is found to have been moderate, the influence of profits and, in the case of residential building, of the shortage and abundance of houses [1] being much larger (equations (2.4) and (2.5)).

(*c*) Although statistics are incomplete, it is nevertheless probable that movements in commodity stocks were dependent only in a small degree on interest rates (equation (2.6)).

(*d*) Conclusions (*a*) and (*c*) are confirmed by the results found for short-term loans, which also seem to have depended very little on the short-term rate of interest (equation (4.3)).

(*e*) The supply of short-term credits seems to have been fairly elastic (equation (4.56), in combination with equation (4.63)).

(*f*) Evidence of a change in the attitude with regard to rationing of credit, apart from the use of interest rates, is not easily found; for neither investment activity nor the demand for loans shows in 1929 any abnormalities in its dependence on its causes. In the event of deliberate rationing, we should expect actual investment activity and new loans to stand below the levels prescribed by their "demand factors" (*cf.* equations (2.4) and (4.3)).

Thus, the general impression is that the monetary system has been elastic. This means that no large influence has been exerted by monetary hindrances on the effects of other factors,

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[1] This fact seems, even more than others, to be peculiar to the United States.

so that these other factors have been allowed to work out fairly completely. Thus the evidence does not seem to support any view according to which influences in the field of money are the chief factors in the business cycles considered.[1] Only if interest rates had showed much larger fluctuations than they actually did would their influence have been important. This does not of course imply that a direct attempt to increase the money value of total demand — *e.g.*, by Government spending — would not be important. It did not, however, occur in a large degree in the period studied, and the cycles found must be explained otherwise.

For the period studied, only small traces are found of the tendencies emphasised by Mr. HAWTREY.[2] The proportion of wages to other incomes is only very slightly changed, and the amount of legal money in circulation did not increase in 1928 and 1929; nor were the limits of the note issue reached.

A fairly considerable influence, however, was found to be exercised by hoarding in the following years, as a consequence of the severe depression. This influence, which acted through interest rates on share prices, and from them on consumption (equations (2.1), (5.3)) and investment (equations (2.4), (1.9)), seems to be the most important from the monetary sphere.

(7.4) NON-MONETARY OVER-INVESTMENT THEORIES

We may now turn to some of the best-known *non-monetary theories* and the factors they use in the explanation of the cycle. First the question of *over-investment* may be examined.

Professor HABERLER describes over-investment as a "vertical disequilibrium or maladjustment" in the structure of production — *i.e.*, a situation in which industries in the higher stages of production are over-developed relatively to those in the lower stages. The supporters of the over-investment school maintain that such a situation arises during the upswing.

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[1] *Cf.* HABERLER, *loc. cit.*, Chapters 2 and 3 A.

[2] *Cf.* HABERLER, *loc. cit.*, Chapter 2. It may be remembered that Mr. HAWTREY doubts whether an ordinary business cycle has shown itself in that period.

In accordance with this view, our equations show that the higher the value of Z (general profits), the higher is the ratio capital goods production bears to consumers' goods production. This fact certainly plays a rôle in the cumulative process: the greater Z is, the greater, a little later, becomes investment activity; and the greater the latter, the greater Z is at the same moment, because of the higher general activity. This process is very clearly shown in the equations, which state, on the basis of our calculations, that profits are a highly important factor for investment activity.

One special form in which the over-investment theories have sometimes — and especially in the last few years — been formulated is that of the *acceleration principle* — *i.e.*, that fluctuations in investment would be chiefly governed by the rate of increase in consumers' goods production.

> " The proposition that changes in demand for consumers' goods are transmitted with increasing intensity to the higher stages of production serves, in conjunction with other factors which have already been mentioned, as an explanation of the cumulative force and self-sustaining nature of the upward movement. . . . The matter is of the greatest practical importance for the reason that much light is shed on the fact, which in the last few years has been more and more recognised and emphasised, that it is the production of *durable* goods, of consumers' goods as well as of capital goods, which fluctuates most violently during the business cycle." [1]

This principle we have not found to be of much importance, at least so far as a *direct* influence on the *shorter* fluctuations of investment activity is concerned (equation (2.4)). It must not be overlooked, however, that there is a high intercorrelation between the production of consumers' goods and that of investment goods; but, even in countries where this parallelism was not found to exist, we found only little direct influence of the rate of increase in consumers' goods production.[2]

Over-investment is attributed by several authors to the capitalistic structure of production, and especially to the long *period required for the construction of physical capital*.[3]

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[1] Haberler, *loc. cit.*, pages 86-87.

[2] *Cf.* Vol. I, Chapters III and V.

[3] *Cf.* Haberler, *loc. cit.*, pages 134-136.

In our equations, the *construction period* plays a very definite and also a rather important rôle. For houses, it is one of the causes — but here only a minor one — of the duration of the cycle. For general investment activity, the existence of a lag of about half a year has a clear influence on the damping ratio and the length of the cycles. This can be seen by changing the lag between profits and investment: this changes considerably the coefficients in the "final equation" which determine the cycles (*cf.* section (6.9)). The relation between the construction period and the period of the cycle is very complicated; at any rate, it does not follow from our calculations that cycles would be abolished, were there no lag in equation (2.4).

Before leaving these theories, a word may be said about the order of the revival in consumers' goods production and producers' goods production respectively. A good deal of attention is given to this question by SPIETHOFF,[1] CASSEL, MITCHELL, and others, and they all hold the opinion that capital goods show the cycle before consumers' goods. Statistically, no evidence of any systematic lag or lead is found, either in the United States after the war, or in a number of other countries.

(7.5) CHANGES IN COSTS

The *element of changing costs of production*, which has sometimes been stressed as a cause of crises,[2] seems to have been

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[1] *Cf.* HABERLER, *loc. cit.*, pages 78-79:

"The phenomenon (alleged to be frequent) of consumers' goods industries feeling the setback of the depression much later than the capital-goods industry is regarded as a verification of the [over-investment] theory."

[2] *Cf.* MITCHELL, quoted in HABERLER, *loc. cit.*, pages 107-108.

"The decline in overhead cost per unit of output [which was brought about by the first increase in production after the trough of the depression] ceases when enterprises have once secured all the business they can handle with their standard equipment, and a slow increase of these costs begins when the expiration of the old contracts makes necessary renewals at the high rates of interest, rent, and salaries which prevail in prosperity. Meanwhile, the operating costs rise at a relatively rapid rate. Equipment which is antiquated and plants which are ill located or otherwise work at some disof less importance. This may be supported by the following evidence.

(i) In so far as higher costs mean higher wages, they are also, if really paid, at almost the same moment, higher incomes, and in the balance of total profits they almost cancel out (relations (5.10), (2.1), (1.11)). This does not of course apply to a country with a large international trade; but the United States is to a high degree a "closed economy". Nor does it apply to those higher costs which are not paid out but which prevent production from taking place. In this connection, however, the conclusions (ii) and (iii) are of importance.

(ii) The demand for investment goods seems to be rather inelastic with regard to price; and in any case the adverse influence of a high price will as a rule, and partly as a consequence, be considerably outweighed by the favourable influence of profits occurring usually at the same time (equation (2.4)).

(iii) Consumption expenditure is also not influenced unfavourably, but rather favourably, by a rise in prices (equation (2.1)).

In short, there has been a tendency for moderate increases in costs to lift all money values to a higher level, rather than to upset the equilibrium. Equilibrium is only upset if prices go up much more than they did in 1929, as they did for instance in 1920 and in some pre-war cycles.

(7.6) OVER-INVESTMENT *vs.* UNDER-CONSUMPTION THEORIES

Under-consumption theories are, in a sense, the opposite of over-investment theories. Professor HABERLER summarises

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advantage are again brought into operation. The price of labour rises, not only because the standard rates of wages go up, but also because of the prevalence of higher pay for overtime. Still more serious is the fact that the efficiency of labour declines, because overtime brings weariness, because of the employment of 'undesirables', and because crews cannot be driven at top speed when jobs are more numerous than men to fill them. The prices of raw materials continue to rise faster, on the average, than the selling prices of products. Finally, the numerous small wastes incident to the conduct of business enterprises creep up when managers are hurried by a press of orders demanding prompt delivery."

as follows their divergent conclusions:

> "Is the turn from prosperity to depression brought about by a shortage of capital or by an insufficiency of the demand for consumers' goods ? Does the investment boom collapse because the supply of capital becomes too small to complete the new roundabout methods of production, or because consumers' demand is insufficient to sustain the increased productive capacity ?
>
> . . . .
>
> "Both theories contemplate what we have called a *vertical* maladjustment in the structure of production; but these vertical maladjustments are not of the same order. As we shall see at once, the 'top ' of the structure of production according to the one theory, the 'bottom ' according to the other, is over-developed in relation to the flow of money. In a sense, both theories can be described as over-investment theories. In the one case, new investments are excessive in relation to the supply of saving; in the other case, they are excessive in relation to the demand for the product. That the distinction is important may be seen from the fact that the conclusions drawn as to the appropriate policy to follow in order to avert, mitigate or postpone the breakdown are diametrically opposed. According to the one view, every measure that tends to increase consumers' demand and to reduce saving is helpful. According to the other view, exactly the opposite policy is called for." [1]

When putting the crucial question with regard to the situation that prevailed in the United States in the year 1929, one circumstance of importance stands out. The over-investment theories are based on the hypothesis of full employment of all capital goods, a situation which may have been approximately realised in some pre-war boom years. It was, however, far from existing in 1929.[2] For this reason, it is highly doubtful

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[1] *Cf.* Haberler, *loc. cit.*, pages 128-129.

[2] This is reflected in our equations by the absence of any bottleneck — even in capital-goods industries — in 1929 (equations (3.3) and (3.5)). The lack of capacity figures covering a representative part of industry makes it impossible to indicate how far production could still have risen in 1929 before a scarcity of capital goods would have developed.

whether the over-investment theory was applicable to that situation. There is a further reason — viz.: the elasticity of the credit system (*cf.* section (7.3)), even in 1929 — which makes it probable that a deficiency of savings, if it had happened, could easily have been remedied by the use of additional credits.[1] In other words, if more had been saved in 1929, it would have led to such a slight fall in interest rates that investment activity would hardly have been stimulated; and the loss of this amount of extra saving in the market for consumers' goods would probably not have been compensated.

(7.7) AGRICULTURAL THEORIES

Finally, some attention may be given to *agricultural theories.*

Professor HABERLER distinguishes between the influences exerted by agriculture — *i.e.*, by changing harvests — on general business conditions, and the influences exerted by general business conditions on agriculture.[2]

The influence of irregularities in harvests on general business conditions shows itself in the determination of farm prices (equation (3.4)) and, consequently, on general prices (equation (3.3)), as well as in the influence of farm prices on consumption (equations (2.2), (2.3) and (2.1)). Farm prices themselves are rather strongly affected by supply fluctuations (the flexibility being about 2); but it seems doubtful whether the influence of f on the system as a whole is large. This doubt

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[1] *Cf.* G. CASSEL, reproduced in HABERLER, *loc. cit.*, page 79.

"The typical modern trade boom does not mean over-production, or an over-estimate of the demands of the consumers or the needs of the community for the services of fixed capital, but an over-estimate of the supply of capital, or of the amount of savings available for taking over the real capital produced. What is really over-estimated is the capacity of the capitalists to provide savings in sufficient quantity."

[2] *Cf.* HABERLER, *loc. cit.*, page 154.

is primarily based upon the following evidence:

(i) Farm prices fluctuate chiefly because of changes in demand; the influence of L_w+L_s in equation (3.4) is much larger than that of f.

(ii) The fluctuations in farm prices are only to an extent of 20% reflected in the fluctuations of prices of finished consumers' goods and services (equations (1.8) and (3.3)).

(iii) The rôle of prices in the business cycle is restricted for reasons given above (section (7.5).

As to the influence exercised on agricultural incomes by fluctuations in industrial activity accompanied by similar fluctuations in money demand in general, Professor HABERLER remarks that "the process is tempered by two factors:

> "(1) The demand for consumers' goods as a whole is more stable than the demand for all goods;
>
> "(2) The demand for consumers' goods of agricultural origin is more stable than that for consumers' goods as a whole."[1]

The influence of general business conditions on farm prices is reflected by the term $2.61(L_w+L_s)$ in equation (3.4). This figure points to an income elasticity for expenditure on agricultural goods of about 0.5 (*cf.* section (3.4)), whereas we found the income elasticity for total consumption to be in the neighbourhood of 0.9;[2] these findings are in accordance with Professor HABERLER's second point. The first point is equally confirmed by our figures.

In commenting on the various "agricultural" theories, Professor HABERLER observes:[3]

> "It is a more serious shortcoming of these 'agricultural' theories that they are not agreed on the important point as to whether

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[1] HABERLER, *loc. cit.*, pages 165-166.

[2] A weighted average of the marginal propensities to consume with respect to urban labour and non-labour income, and farmers' income, divided by U'/average income.

[3] HABERLER, *loc. cit.*, page 154.

> plentiful harvests are correlated with prosperity and poor harvests with depression, or the other way round; and their divergence in this respect is symptomatic of a fundamental disagreement as to the channels by which the influence of agricultural fluctuations is brought to bear on other departments of economic life.''

In this connection, it may be pointed out that, in our final equation for Z^c, various terms occur representing the influence of autonomous changes in harvest (f), the first and largest with a negative, the second with a positive sign ($-1.847f + 0.708f_{-1}$). And it is quite probable that, in any final equation obtained for other variables, these terms will again be different from those in the equation for Z^c. All this reflects the fact that harvest fluctuations work in a complicated way, partly positively, partly negatively.

(7.8) SOME GENERAL STATEMENTS ON THE CHARACTER OF THE CYCLE

This set of observations on some of the more important business-cycle theories may be concluded by a consideration of certain very general statements made by a number of different authors on the character of cyclical movements.

1. The first is that the *depression is an inevitable consequence and a necessary readjustment of certain disproportionalities* which have previously developed.[1] Our statistical investigations show that, with the given economic structure (described by the coefficients in our elementary equations), the depression

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[1] *Cf.* HABERLER, *loc. cit.*, pages 57-58:

"The depression was originally conceived of by the authors of the monetary over-investment school as a process of adjustment of the structure of production, and was explained in non-monetary forms. During the boom, they argued, the process of production is unduly elongated. This elongation has accordingly to be removed and the structure of production has to be shortened or, alternatively, expenditure on consumers' goods must be reduced (by retrenchment of wages and other incomes which are likely to be spent wholly or mainly on consumers' goods) sufficiently to make the new structure of production possible. This involves a lengthy and painful process of rearrangement."

is certainly a consequence of the preceding boom. It is necessary, however, only in so far as (i) the economic structure is not changed and (ii) no exogenous shocks (amongst which certain measures of policy are to be counted) occur. Several forms of policy seem to be possible which would prevent a depression from developing and yet overcome the disproportionalities.

2. A second proposition is *that there may occur an automatic revival from a depression.*[1] The mechanism found for the United States is such that an automatic revival, indeed, is to be expected for the short waves: the movements were found to be cyclical (*cf.* Chapter VI). As to movements of longer duration, we are not yet able to make a definite statement (*cf.* page 149).

3. A third statement made by a number of theorists is that "*the recovery from the depth of the depression has a wrong twist from the beginning*". This statement must probably be understood in the sense that it is impossible to prevent a boom if once recovery has started from the bottom. In this sense it is the counterpart of the above statement 1, and seems untenable on the same grounds. This has been shown explicitly in section (6.3); and, since this demonstration is independent of any particular features of the system of equations, it may as well be formulated in this non-mathematical way: that the position in any year, though depending in part on what happened before, may be considerably influenced by fresh "shocks"; and, if such shocks are a systematic set of measures, it is certainly within the possibilities to prevent a boom from developing to dangerous heights.

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[1] *Cf.* HABERLER, *loc. cit.*, page 391.

APPENDIX A

LIST OF VARIABLES INCLUDED IN SYSTEM

(See inset at end of volume.)

APPENDIX B

TABLE I. — LIST OF EQUATIONS

1.1 $A = 1.50\,C^i + 0.90\,B^i + 0.84\,n - 18.0\,m_{Lb}$
1.2 $d = 1.25\,D - 0.11\,t$
1.3 $E = D + L_c + K_I + K_{R'} + (E_E - E'_F - E''_F)$
1.4 $h = h_{-1} + 0.92\,v_B$ or $v_B = 1.09\,\Delta h$
1.5 $M = M'' + M'$
1.6 $M = Bs + 0.9\,B^b - 2.93\,m_{Lb}$
1.7 $m_{Ls} = 0.67\,d - 0.041\,n$
1.8 $p = 0.80\,p' + 0.20\,m_R$
1.9 $S = \int (Z^c - D) + 2.29\,t$
1.10 $U = u + 0.60\,p$
1.11 $u = u' + w - w_{-1}$
1.12 $U' = u' + 0.60\,p$
1.13 $V = V' + V_B$
1.14 $v = v' + v_B$
1.15 $V' = v' + 0.15q$
1.16 $V_B = 0.98\,v_B + 0.028q_B$

2.1 $U' = 0.95\,(L_w + L_s) + 0.77\,E + E'_F + 0.28\,G + 0.049\,\Delta\,p^f + 0.03\,p + 0.37\,t$
2.2 $E'_F = 0.025\,p^f$
2.3 $E''_F = 0.015\,p^f$
2.4 $v' = 0.33\,(Z^c + Z^c_{-1}) - 0.47\,[m_{Ls} + (m_{Ls})_{-1}] - 0.015\,(q + q_{-1}) + 0.060\,[p + p_{-1} - \frac{1}{2}\,l - \frac{1}{2}\,l_{-1}] + 0.63\,t$
2.5 $v_B = -0.30\,h_{-4} + 0.074\,Z^c + 0.042\,m_R - 0.031\,q_B - 0.038\,m_{Lb} + 0.10\,t$
2.6 $w = 0.105\,u' + 0.047\,u'_{-1} - 0.187\,(m_S)_{+\frac{1}{2}} - 0.307\,t$

3.1 $l_{+0.42} = 0.30\,(u + v) + 0.39\,p + 0.51\,t$
3.2 $m_{\mathrm{R}} = -3.51\,h_{-2} + 2.13\,(\mathrm{L}_w + \mathrm{L}_s)_{-1} - 0.25\,p'_{-1} + 1.21\,t$
3.3 $\dot{p}_{+0.21} = 0.47\,l + 0.25\,p^f - 1.04\,t$
3.4 $p^f = -9.54\,f + 2.61\,(\mathrm{L}_w + \mathrm{L}_s) - 0.58\,t$
3.5 $q = 0.35\,l + 1.29\,v'_{-0.46} - 2.58\,t$

4.1 $\mathrm{B} = 4.88\,t$
4.2 $\mathrm{C} = 3.18\,t$
4.3 $\mathrm{B}_s = 0.16\,(\mathrm{U} + \mathrm{V}) + 0.26\,m_{\mathrm{L}s} + 2.56\,m_{\mathrm{L}b} + 0.055\,n + 0.08\,\mathrm{C}$
4.4 $\mathrm{M} = 6.6\,m_{\mathrm{S}} - 25\,\mathrm{M}' - 26\,\mathrm{VC} + 26\,\mathrm{A}u + 26\,\mathrm{P}$
4.56 $\mathrm{B}_s = 0.63\,\mathrm{M} + 1.51\,m_{\mathrm{S}} - 1.10\,m_{\mathrm{L}b} + 0.12\,\dot{m}_{\mathrm{L}b}$
4.57 $\mathrm{B}^b = 0.41\,\mathrm{M} - 1.68\,m_{\mathrm{S}} + 4.48\,m_{\mathrm{L}b} - 0.14\,\dot{m}_{\mathrm{L}b}$
4.61 $\mathrm{M}' = 0.043\,(\mathrm{L}_w + \mathrm{L}_s + \mathrm{E}'_{\mathrm{F}}) - 0.076\,t + \mathrm{H}$
4.612 $\mathrm{H} = 0.30\,(\mathrm{Z}^c_m - \mathrm{Z}^c - 7)''$
4.63 $\mathrm{M}'' = 0.29\,(\mathrm{U} + \mathrm{V}) + 0.03\,\mathrm{C} + 0.020\,n - 0.42\,m_{\mathrm{S}} + 0.90\,t$
4.8 $n = 20.6\,d - 6.4\,m_{\mathrm{L}b} + 2.36\,(n - n_{-1} - 20)'' + 2.09\,t - 5$

5.1 $\mathrm{D} = 0.151\,\mathrm{Z}^c + 0.083\,\mathrm{Z}^c_{-1} + 0.075\,\mathrm{S}_{-1}$
5.2 $\mathrm{E}_{\mathrm{E}} = \mathrm{E}'_{\mathrm{F}} + \mathrm{E}''_{\mathrm{F}} + 0.110\,\mathrm{Z}^c + 0.066\,\mathrm{Z}^c_{-1} + 0.16\,t$
5.3 $\mathrm{G} = 0.088\,\dot{n} + 0.112\,\dot{n}_{-1}$
5.4 $\mathrm{K}_{\mathrm{I}} = 0.02 \int (\mathrm{Z}^c_{-1} + \mathrm{Z}^c_{-2}) + 0.11\,t$
5.5 $\mathrm{K}_{\mathrm{R}} = 0.069\,\mathrm{Z}^c + 0.029 m_{\mathrm{R}}$
5.6 $\mathrm{L}_c = 0.047\,\mathrm{Z}^c + 0.046\,\mathrm{Z}^c_{-1} + 0.073\,t$
5.7 $\mathrm{L}_s = 0.170\,\mathrm{Z}^c + 0.185\,\mathrm{Z}^c_{-1} + 0.225\,\mathrm{Z}^c_{-2} + 0.40\,t$
5.8 $\mathrm{L}_w = 0.28\,(u + v) + 0.30\,l - 0.73\,t$
5.9 $\mathrm{N} = 0.036\,(u + v) + 0.037\,q + 0.04 \int \mathrm{V} + 0.12\,t$
5.10 $\mathrm{Z} = \mathrm{U} + \mathrm{V} - (\mathrm{L}_w + \mathrm{L}_s + \mathrm{L}_c + \mathrm{K}_{\mathrm{R}} + \mathrm{K}_{\mathrm{I}} + \mathrm{N})$
5.11 $\mathrm{Z} = 1.45\,\mathrm{Z}^c + 0.26\,\mathrm{Z}^c_{-1} - 0.02\,t$

TABLE II. — TREATMENT OF THE MONETARY GROUP

(All trend terms omitted)

No.	References	Variable "explained"	Explanatory variables								Omissions
			$Au + P$	M'	m_S	n	$U + V$	C	m_{Lb}	M	
1	4.4	M =	26	− 25	6.6						VC [*cf.* section (4.62)]
2	4.63, 1.5	M =		1	− 0.42	0.020	0.29	0.03			
3	1 − 2	0 =	26	− 26	7.02	− 0.020	− 0.29	− 0.03			
4	3	$-0.42\ m_S$ =	1.56	− 1.56		− 0.001	− 0.02				
5	2 + 4	M =	1.56	− 0.56		0.019	0.27	0.03			
6	4.56	B_s =			1.51				− 1.10	0.63	$0.12\ \dot{m}_{Lb}$
7	5	0.63 M =	0.98	− 0.35		0.012	0.17	0.02			
8	3	$1.51\ m_S$ =	− 5.61	5.61		0.004	0.06	0.01			
9	6 + 7 + 8	B_s =	− 4.63	5.26		0.016	0.23	0.03	− 1.10		
					m_{Ls}					d	
10	4.3	B_s =			0.26	0.055	0.16	0.08	2.56		
11	9 − 10	0 =	− 4.63	5.26	− 0.26	− 0.039	0.07	− 0.05	− 3.66		
12	1.7, 4.2	0 =			0.26	0.011		0.05		− 0.17	
13	11 + 12	0 =	− 4.63	5.26		− 0.028	0.07		− 3.66	− 0.17	
					$(n - n_{-1} - 20)''$	$L_w + L_s + E_F'$		H			
14	4.8	$-0.028n$ =			− 0.07				0.18	− 0.58	Constant term
15	13 + 14	0 =	− 4.63	5.26	− 0.07		0.07		− 3.48	− 0.75	
16	15	m_{Lb} =	− 1.33	1.51	− 0.02		0.02			− 0.22	
17	16	$-6.4\,m_{Lb}$ =	8.51	− 9.67	0.13		− 0.13			1.38	
18	4.8, 16	n =	8.51	− 9.67	2.49		− 0.13			22.0	Constant term
19	4.61	1.51 M′ =				0.065		1.51			
20	4.61	− 9.67 M′ =				− 0.416		− 9.67			
21 = 4.92	16 + 19	m_{Lb} =	− 1.33		− 0.02	0.065	0.02	1.51		− 0.22	
22 = 4.91	18 + 20	n =	8.51		2.49	− 0.416	− 0.13	− 9.67		22.0	

TABLE III. — THE PRICE GROUP AND THE STRATEGIC GROUP

(All trend and cumulants omitted)

No.	References	Equations used	Omissions
	Price group		
1.8′	1.8	$p = 0.80\,p' + 0.20\,m_{\mathrm{R}}$	
1.10′	1.10	$\mathrm{U} = u + 0.60\,p$	
1.11′	1.11	$u = u' + w - w_{-1}$	
1.12′	1.12	$\mathrm{U}' = u' + 0.60\,p$	
3.1′	3.1	$l_{+0.42} = 0.30\,(u + v) + 0.39\,p$	
3.2′	3.2, 5.7	$m_{\mathrm{R}} = -3.51\,h_{-2} + 0.362\,\mathrm{Z}^c_{-1} + 0.394\,\mathrm{Z}^c_{-2} + 0.479\,\mathrm{Z}^c_{-3} + 2.13\,\mathrm{L}_{w-1}$ $- 0.25\,p'_{-1}$	
3.3′	3.3	$p'_{+0.21} = 0.47\,l + 0.25\,p^f$	
3.4′	3.4, 5.7	$p^f = -9.54\,f + 0.444\,\mathrm{Z}^c + 0.483\,\mathrm{Z}^c_{-1} + 0.587\,\mathrm{Z}^c_{-2} + 2.61\,\mathrm{L}_w$	
3.5′	3.5	$q = 0.35\,l + 1.29\,v'_{-0.46}$	
5.8′	5.8	$\mathrm{L}_w = 0.28\,(u + v) + 0.30\,l$	

TABLE III *(continued)*

No.	References	Equations used	Omissions
	Strategic group		
2.1′	5.1, 2.2, 5.7, 1.3, 5.3, 5.1, 1.9, 5.6, 5.4, 5.5, 5.2	$U' = 0.452\ Z^c + 0.326\ Z^c_{-1} + 0.214\ Z^c_{-2} + 0.056\ (n - n_{-1})^*$ $+ 0.049\ \Delta p^f + 0.95\ L_w + 0.022\ m_R + 0.025\ p^f + 0.03\ p$	
2.4′	2.4, 1.7, 1.2, 5.1, 1.9	$v' = 0.271\ Z^c + 0.237\ Z_{-1} - 0.033\ Z^c_{-2} + 0.020\ n + 0.019\ n_{-1}$ $- 0.030\ q_{-0.50} + 0.120\ (p - \tfrac{1}{2} l)_{-0.50}$	
2.5′	2.5	$\dagger\ v_B = - 0.30\ h_{-4}$	$0.074\ Z^c, 0.042\ m_R, - 0.031\ q_B,$ $- 0.038\ m_{Lb}$ †
1.4′	1.4	$v_B = 1.09\ \Delta h$	
2.6′	2.6	$w = 0.105\ u' + 0.047\ u'_{-1}$	$- 0.187\ (m_S)_{+\frac{1}{2}}$
4.6′	4.612	$H = 0.30\ (Z^c_m - Z^c - 7)''$	
4.91′	4.91, 1.2, 5.1, 1.9, 2.2, 5.7, 1.13, 1.14, 1.15, 1.16	$n = 4.087\ Z^c + 2.210\ Z^c_{-1} - 0.095\ Z^c_{-2} - 0.13\ (U + v) - 0.020\ q$ $- 0.42\ L_w - 0.011\ p^f + 2.49\ (n - n_{-1} - 20)'' - 9.67\ H$ $+ 8.51\ (Au + P)$	$- 0.004\ q_B$
5.10′	5.10, 5.11, 1.13, 1.15, 1.16, 2.5, 5.4 — 5.7, 5.9	$1.45\ Z^c + 0.26\ Z^c_{-1} = U + v' - 0.29\ h_{-4} + 0.11\ q + 0.012\ m_R - L_w$ $- 0.036\ (u + v) - 0.213\ Z^c - 0.231\ Z^c_{-1} - 0.225\ Z^c_{-2}$	$- 0.002\ q_B, \quad - 0.037\ m_{Lb}$

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* $0.025\ \dot{n} + 0.031\ \dot{n}_{-1}$ may, with good approximation, be replaced by $0.056\ \dot{n}_{-\frac{1}{2}}$, which again may be replaced by $0.056\ (n - n_{-1})$.

† The approximating form of equation 2.5′ will only be used:

(i) to study the systematic connection between 2.5 and 1.4 (section (6.6)).

(ii) in the substitution of v_B in line 265, where v_B has a very small coefficient.

In equation 5.10′, however, v_B has been substituted by equation 2.5, with the omission only of the m_{Lb}-term.

TABLE IV. — TREATMENT OF THE PRICE GROUP

No.	References[1]	Variable "explained"	Explanatory variables								
			Coeff.	Var.	Lag	Coeff.	Lag	Coeff.	Lag	Coeff.	Lag
			Various			m_R		p^f		$u + v$	
101	1.8′, 3.3′	$p =$	0.376	l	−0.21	0.200		0.200	−0.21		
102	101, 3.1′	$p =$	0.147	p	−0.63	0.200		0.200	−0.21	0.113	−0.63
103	102, R	$p =$				0.234	−0.11	0.234	−0.32	0.132	−0.74
										$U + v$	
104	103, 1.10′	$p =$	−0.079	p	−0.74	0.234	−0.11	0.234	−0.32	0.132	−0.74
105	104, R	$p =$				0.217	−0.06	0.217	−0.27	0.122	−0.69
106	1.10′, 105, C, R	$(u+v) =$				−0.130	−0.06	−0.130	−0.27	0.927	+0.05
107	3.1′, 106, 105, C	$l =$				0.046	−0.48	0.046	−0.69	0.326	−0.48
108	5.8′, 106, 107, C	$L_w =$				−0.022	0.23	−0.022		0.358	−0.09
109	3.5′, 107, C	$q =$	1.290	v'	−0.46	0.016	−0.48	0.016	−0.69	0.114	−0.48
								$-9.54\,f + 0.444\,Z^c + 0.483\,Z^c_{-1} + 0.587\,Z^c_{-2}$			
110	3.4′, 108, R	$p^f =$				−0.054	0.23	0.946		0.884	−0.09
						$-3.51\,h_{-2} + 0.362\,Z^c_{-1} + 0.394\,Z^c_{-2} + 0.479\,Z^c_{-3}$					
111	3.2′, 3.3′, 107, 108, 110, C, R	$m_R =$				0.956	0.04	−0.104	−1.12	0.596	−0.99
112	110, 111, C, R	$p^f =$				−0.052	0.27	0.952		0.852	−0.07

We now combine, in 111 and in 112, the first two terms and put:

$$F^1 = -3.36\,h_{-1.96} + 0.021\,Z^c + 0.299\,Z^c_{-1} + 0.330\,Z^c_{-2} + 0.373\,Z^c_{-3} + 0.992\,f_{-1.12}$$
$$F^2 = 0.181\,h_{-1.73} + 0.415\,Z^c + 0.441\,Z^c_{-1} + 0.536\,Z^c_{-2} - 0.015\,Z^c_{-3} - 9.084\,f_{-0.01}$$

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[1] See note on next page.

TABLE IV *(continued)*

No.	References [1]	Variable "explained"	Explanatory variables								
			Coeff.	Var.	Lag	Coeff.	Lag	Coeff.	Lag	Coeff.	Lag
			Various			**F1**		**F2**		**U + v**	
113	111, F¹	m_R =				1.000				0.596	*—0.99*
114	112, F²	p^f =						1.000		0.852	*—0.07*
115	105	p =				0.217	*—0.06*	0.217	*—0.27*	0.436	*—0.65*
116	106 } 113, 114, C	$u+v$ =				— 0.130	*—0.06*	— 0.130	*—0.27*	0.739	*0.22*
117	107	l =				0.046	*—0.48*	0.046	*—0.69*	0.392	*—0.58*
118	108	L_w =				— 0.022	*0.23*	— 0.022		0.326	*—0.06*
119	109	q =	1.290	v'	*—0.46*	0.016	*—0.48*	0.016	*—0.69*	0.138	*—0.59*

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[1] The symbols C and R are used in the column "References" to indicate:

C: a *combination* into one term of two or more terms with the same variable on the right-hand side of the equation. The coefficient of this one term is found by the addition of the coefficients of the various terms combined, and its lag is an average of the lag of the various terms weighted according to the coefficients of each of those terms:

$$\beta_1 X_{-\theta_1} + \beta_2 X_{-\theta_2} + \dots = (\beta_1 + \beta_2 + \dots) X_{-\frac{\beta_1\theta_1 + \beta_2\theta_2 + \dots}{\beta_1 + \beta_2 + \dots}}$$

R: a *rearrangement*, consisting in the combination as described above, on the left-hand side, of terms with the same variable in both members of the equation, and the subsequent application to all terms of such a factor and such a shift in time as to obtain a coefficient 1 and a lag 0 in the left-hand member.

TABLE V. — TREATMENT OF THE STRATEGIC GROUP

In the course of the following elimination process, three more F's will be introduced:

$$F^3 = 0.452\,Z^e + 0.326\,Z^c_{-1} + 0.214\,Z^e_{-2} + 0.056\,(n - n_{-1}) + 0.049\,\Delta p^f \qquad cf.\ \text{equation } (2.1')$$
$$F^4 = 0.271\,Z^c + 0.237\,Z^c_{-1} - 0.033\,Z^e_{-2} + 0.020\,n + 0.019\,n_{-1} \qquad cf.\ \text{equation } (2.4')$$
$$F^5 = F^3 - 0.049\,\Delta p^f$$

In some cases, terms with coefficients of 0.001 or 0.002 have been omitted. Exceptionally, terms with larger coefficients have been omitted after it had been ascertained that their repercussions were very weak.

No.	References	Variable "explained"	Explanatory variables																						No.
201	1.11′, 2.6′	$u =$	$1.105\,u' - 0.058\,u'_{-1} - 0.047\,u'_{-2}$																						201
		Variables →	$0.001\ (U+\nu)$				$0.001\ F^1$			$0.001\ F^2$				$0.001\ F^3$				$0.001\ \nu_B$		$0.001\ \nu$				$0.001\ F^4$	
		Lags →	0	−1	−2	−3	0	−1	−2	0	−1	−2	−3	0	−1	−2	−3	0	−1	0	−1	−2	−3	0	
202	118	$0.95\,L_w =$	291	19			−26	5		−21															202
203	115	$-0.57\,p =$	−87	−162			−117	−7		−91	−33														203
204	114	$0.025\,pf =$	20	1						25															204
205	113	$0.022\,m_R =$		13			22																		205
206	+, 2.1′, 1.12	$u' =$	224	−129			−121	−2		−87	−33			1000											206
207	206	$0.105\,u' =$	24	−14			−13			−9	−3			105											207
208	206	$-0.058\,u'_{-1} =$		−13	7			7			5	2			−58										208
209	206	$-0.047\,u'_{-2} =$			−11	6			6			4				−47									209
210	115	$0.60\,p =$	92	170			122	8		95	35														210
211	+, 1.10, 201	$U =$	340	14	−4	6	−12	13	6	−1	4	6		1105	−58	−47				515					211
212	211, R	$U =$		21	−6	9	−18	20	9	−2	6	9		1674	−88	−71									212
213	212	$0.021\,(U+v)_{-1} =$													35	−2	−1				32				213
214	212	$-0.006\,(U+v)_{-2} =$														−10	1					−9			214
215	212	$0.009\,(U+v)_{-3} =$															15						14		215
216	(212+ . . . 215)	$U =$					−18	20	9	−2	6	9		1674	−53	−83	15			515	32	−9	14		216
217	119, 1.14	$-0.030\,q_{-0.50} =$		−4														2	37	−2	−37				217
218	115	$0.120\,p_{-0.50} =$		44	8		11	15		6	20														218
219	117	$-0.060\,l_{-0.50} =$		−22	−2			−3			−2	−1													219
220	+, 2.4′	$v' =$		18	6		11	12		6	18	−1						2	37	−2	−37			1000	220
221	212	$0.018\,(U+v)_{-1} + 0.006\,(U+v)_{-2} =$													30	9					27	10			221
222	220+221	$v' =$					11	12		6	18	−1			30	9		2	37	−2	−10	10		1000	222
			$0.001\ F^4$																						
223	222, 1.14	$-0.002\,v - 0.010_{-1}\,v + 0.010_{-2} =$	−2	−10	10													−2	−10						223
224	222+223	$v' =$	998	−10	10		11	12		6	18	−1			30	9			27						224
225	216, 224, 1.14	$U =$	514	27	−4	14	−12	26	9	1	15	9		1674	−38	−77	15	515	46						225
226	224, 225, 1.14	$(u+v) =$	1512	17	6	14	−1	38	9	7	33	8		1674	−8	−68	15	1515	73						226
														$0.001\ F^5$						$0.001\ p^f$					
227	114, 226, {definition of F⁵	$pf =$	1198	104	6	11	−1	30	9	1006	27	8		1326	94	−54	8	1200	149	65	−60	−8	3		227
228	227, R	$pf =$	1281	111	6	12	−1	32	10	1076	29	9		1418	101	−58	9	1283	159		−64	−9	3		228
229	228	$-0.064\,pf_{-1} - 0.009\,pf_{-2} + 0.003\,pf_{-3} =$		−82	−19	3			−2		−69	−12			−91	−19	7		−82						229
230	228+229	$pf =$	1281	29	−13	15	−1	32	8	1076	−40	−3		1418	10	−77	16	1283	77						230
231	230, definition of F⁵	$F^3 =$	63	−63				2	−2	53	−53			1069	−69	−4		63	−63						231
232	226, 231	$(u+v) =$	1617	−89	3	19	−1	38	9	100	−56	4	5	1790	−125	−72	21	1620	−33						232
233	224, 231	$v' =$	998	−8	9	−1	11	12		6	20	−3			32	8	−1		29						233
234	119, 232, 233	$q =$	788	712	−6	5	16	25	11	15	34	5	−2	102	160	11	−1	92	149						234
235	118, 232	$L_w =$	495	5	−1	6	−27	17	4	9	−15		2	548	−2	−25	5	496	22						235
236	116, 232	$(u+v) =$	1459	−344	18	17	−123	26	2	−5	−102	13	4	1615	−405	−45	31	1461	−294						236
237	113, 232	$m_R =$	10	953	−53	2	1000	−1	22	1	59	−33	2	11	1055	−74	−42	10	956						237
			Treatment of n-equation and Z^c-equation																						
238	232	$-0.13\,(U+v) =$	−210	12		−2		−5	−1	−13	7	−1	−1	−233	16	9	−3	−211	4						238
239	235 cf.	$-0.42\,L_w =$	−208	−2		−3	11	−7	−2	−4	6		−1	−230	1	11	−2	−208	−9						239
240	230 4.91′	$-0.011\,pf =$	−14							−12				−16		1		−14	−1						240
241	234	$-0.020\,q =$	−16	−14				−1			−1			−2	−3			−2	−3						241
242	+	$S_n = (238 + \ldots\ 241) =$	−448	−4*		−5*	11	−13	−3*	−29	12	−1*	−2*	−481	14	21	−5*	−435	−9						242
243	232, 1.14	$(U+v') =$	1617	−89	3	19	−1	38	9	100	−56	4	5	1790	−125	−72	21	620	−33						243
244	234 cf.	$0.11\,q =$	87	78	−1	1	2	3	1	2	4	1		11	18	1		10	16						244
245	237 5.10′	$0.012\,m_R =$		11	−1		12				1				13	−1	−1		11						245
246	235	$-L_w =$	−495	−5	1	−6	27	−17	−4	−9	15		−2	−548	2	25	−5	−496	−22						246
247	236	$-0.036\,(u+v) =$	−53	12	−1	−1	4	−1			4			−58	15	2	−1	−53	11						247
248	+	$S_z = (243 + \ldots\ 247) =$	1156	7*	1*	13	44	23	6*	93	−32	5*	3*	1195	−77	−45	14	81	−17						248

		Variables	$0.001\ Z^e$					$0.001\ h$			$0.001\ f$		$0.001\ \nu_B$		$(n-n_{-1}-20)''$	H	$Au+P$	$0.001\ n$				
		Lags	0	−1	−2	−3	−4	−2	−3	−4	0	−1	0	−1	0	0	0	0	−1	−2	−3	
249	242, definitions of F's	S_n, terms with $F^1_{0,-1}$ § $=$		3			−5	−37	44			11										249
250		$F^2_{,-1}$ § $=$	−12	−8	−11	6		−5	2		263	−109										250
251		$F^4_0 =$	−121	−106	15													−9	−9			251
252		$F^5_{0,-1,-2} =$	−217	−151	−89	10	4											−27	27			252
253	4.91′	Other terms of 4.91′ $=$	4087	2210	−95								−435	−9	2.49	−9.67	8.51					253
254	+, 4.91′	$1.036\,n =$	3737	1948	−180	16	−1	−42	46		263	−98	−435	−9	2.49	−9.67	8.51		18			254
255	254, R	$n =$	3607	1880	−174	15	−1	−41	44		254	−95	−420	−9	2.40	−9.33	8.21		17			255
256	255	$0.017\,n_{-1} =$		61	32	−3			−1	1		4		−7	†	†	†					256
257	255+256	$n =$	3607	1941	−112	12	−1‡	−41	43	1‡	254	−91	−420	−16	2.40	−9.33	8.21					257
258	248, definitions of F's	S_z, terms with $F^1_{0,-1}$ § $=$	1	13	22	24	9	−148	−77			44										258
259		$F^2_{0,-1}$ § $=$	39	28	36	−18		17	−6		−845	291										259
260		$F^4_{0,-3} =$	313	274	−38	4	3											23	22			260
261		$F^5_{0,-1,-2,-3} =$	540	355	211	−25	−5											67	−71	1	3	261
262	+	$S_z =$	893	670	231	−15	7	−131	−83		−845	335						90	−49	1	3	262
															$(n-n_{-1}-20)''$		H		$Au+P$			
															0	−1	0	−1	0	−1		
263	257, †	n-terms in 262 $=$	325	−2	−104	21	5	−4	6	−2	23	−20	−38	20	0.22	−0.12	−0.84	0.46	0.74	−0.40		263
264	5.10′	Other terms of 5.10′ $=$	−1663	−491	−225					−290			81	−17								264
265	+, 2.5′, 5.10′	$0 =$	−445	177	−98	6	12	−135	−77	−305	−822	315		†	0.22	−0.12	−0.84	0.46	0.74	−0.40		265
266	265, R	$Z^c =$		398	−220	13	27	−303	−173	−685	−1847	708			0.49	−0.27	−1.89	1.03	1.66	−0.90		266

---

* Terms < 0.01 F are neglected in the following calculations.
† Small lagged terms omitted.
§ Lags of terms with h and f rounded to entire number of years.
‡ Neglected.

In some cases, terms with coefficient

No.	References	Variable "explained
201	1.11′, 2.6′	

No.
201

APPENDIX C

STATISTICAL MATERIAL USED

STANDARD UNITS

1. *Money Items:* milliards of \$ (10^9 \$).
2. *Physical Quantities:* milliards of \$ of 1929, *i.e.* values at 1929 prices.
3. *Prices:* Indices, 1929 = 100; except n: 1926 = 100.
4. *Interest rates:* in %.

Variables	Deviations from Average														Average 1919-1932	Variables
	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932		
A	— 76	— 77	— 72	— 51	— 44	— 31	— 10	7	35	76	133	109	37	— 37	174	A
Au	—0.94	—1.20	—0.78	—0.26	—0.01	0.37	0.31	0.39	0.50	0.14	0.22	0.39	0.64	0.17	3.78	Au
B^b	—2.4	—2.6	—2.7	—2.8	—1.9	—1.3	—0.6	—0.5	0.3	1.4	1.3	1.7	3.9	6.8	16.3	B^b
ΔB^e	.	—2.9	1.6	—1.1	1.9	3.2	2.8	2.3	2.6	2.4	1.7	—2.7	—5.0	—7.2	4.6	ΔB^e
$(\Delta B^g+\Delta B^g_s)$	4.81	—1.82	—0.53	—0.68	—0.81	—0.90	—0.70	—1.13	—1.10	—0.67	—0.93	—0.23	1.79	2.95	—0.03	$(\Delta B^g+\Delta B^g_s)$
B^i	— 29	— 24	— 20	— 18	— 13	— 7	— 1	5	10	15	20	23	20	18	100	B^i
(B^i-R^e)	—0.14	0.77	0.66	—0.08	0.09	—0.28	—0.16	—0.08	—0.21	0.19	0.30	—0.38	—0.37	—0.34	0.65	(B^i-R^e)
B_s	—3.8	0.8	—2.4	—5.2	—3.7	—3.0	—0.3	1.4	2.7	5.6	8.0	5.2	0.8	—5.5	36.5	B_s
B^g_s	2.45	1.47	1.26	0.38	—0.76	—0.91	—1.17	—1.29	—1.12	—0.73	—0.48	—0.71	0.24	1.32	2.13	B^g_s
C	— 26	— 20	— 11	— 9	— 6	— 3	— 3	— 2	2	7	15	22	17	12	71	C
ΔC	.	9	2	— 3	3	— 3	— 3	— 1	3	1	10	— 3	— 12	— 5	3	ΔC
H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.89	2.08	—	H
M'	—0.01	0.40	—0.04	—0.33	—0.05	—0.05	—0.09	—0.05	—0.11	—0.16	—0.13	—0.33	0.10	0.84	4.01	M'
M''	—9.7	—7.6	—9.8	—7.4	—4.1	—1.5	1.9	3.9	6.6	9.3	9.4	7.4	4.2	—2.7	43.8	M''
P	0.82	0.74	—0.26	—0.34	—0.38	—0.19	—0.11	—0.16	—0.10	—0.19	—0.39	—0.10	—0.03	0.69	2.28	P
R^r	—0.34	—0.22	—0.39	—0.28	—0.19	—0.04	0.11	0.15	0.23	0.30	0.30	0.31	0.22	—0.16	2.06	R^r
S	— 18	— 13	— 13	— 13	— 8	— 7	— 3	0	6	13	22	22	12	3	35	S

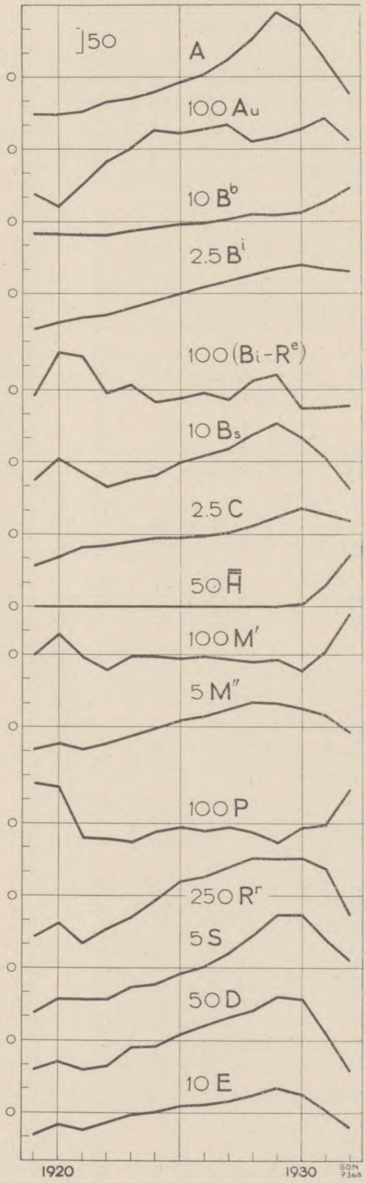
APPENDIX C *(continued)*

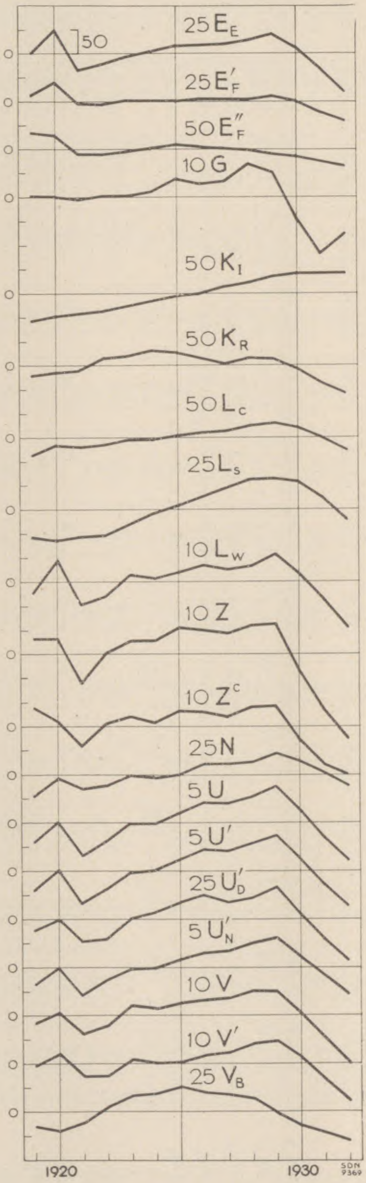
Variables	Deviations from Average														Average 1919-1932	Variables
	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932		
D	—1.2	—0.9	—1.2	—1.1	—0.3	—0.3	0.2	0.6	0.9	1.2	1.8	1.7	0.3	—1.3	4.0	D
E	—4.6	—2.8	—3.7	—2.3	—0.8	—0.1	1.2	1.5	2.2	3.5	4.9	3.7	0.6	—3.0	18.7	E
E_A	—0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.1	E_A
E_E	0.1	1.9	—1.4	—1.0	—0.3	0.1	0.6	0.7	0.8	1.1	1.6	0.5	—1.3	—3.2	12.2	E_E
E'_F	0.5	1.7	—0.1	—0.3	0.1	0.1	0.0	0.2	0.2	0.2	0.4	0.0	—0.9	—1.6	3.7	E'_F
E''_F	0.7	0.6	—0.2	—0.2	—0.1	0.0	0.2	0.1	0.0	0.0	—0.2	—0.3	—0.5	—0.7	1.7	E''_F
G	0.2	0.1	—0.5	0.3	0.4	1.0	3.6	2.6	3.4	6.8	5.2	—4.3	—11.5	—7.5	1.0	G
K_I	—1.1	—0.9	—0.8	—0.7	—0.5	—0.3	—0.1	0.0	0.3	0.5	0.7	0.8	0.8	0.8	3.3	K_I
K_R	—0.4	—0.3	—0.2	0.3	0.4	0.6	0.5	0.3	0.1	0.3	0.3	—0.1	—0.8	—1.2	1.8	K_R
L_c	—0.7	—0.3	—0.4	—0.3	—0.1	—0.1	0.1	0.2	0.3	0.5	0.6	0.4	0.0	—0.6	2.7	L_c
L_s	—2.3	—2.6	—2.3	—2.2	—1.3	—0.3	0.3	1.0	1.8	2.5	2.6	2.3	1.0	—0.9	11.9	L_s
L_w	—2.5	4.1	—4.9	—3.2	1.5	0.5	1.6	3.3	2.5	3.0	5.5	1.5	—3.7	—9.5	27.6	L_w
Z	3.4	3.2	—5.8	0.0	2.8	2.6	5.2	4.8	4.1	5.8	6.3	—3.7	—12.0	—17.1	16.1	Z
Z^c	3.9	1.4	—4.0	0.3	1.8	0.9	3.1	3.0	2.0	3.7	4.2	—2.9	—7.8	—10.1	4.5	Z^c
N	—1.7	—0.3	—1.2	—0.9	0.0	—0.3	0.0	0.9	0.9	1.0	1.7	1.1	0.2	—0.9	9.4	N
U	—7.5	0.1	—13.7	—7.7	—0.3	—0.1	4.0	8.3	8.1	10.7	14.8	4.7	—6.0	—15.7	59.3	U
U'	—8.6	0.0	—13.8	—7.9	—1.0	—0.1	4.0	8.2	7.9	10.8	14.8	5.1	—5.3	—14.7	59.3	U'
U'_D	—1.0	—0.1	—1.9	—1.7	0.0	0.5	1.2	1.9	1.4	1.7	2.6	0.4	—1.6	—3.4	7.5	U'_D
U'_N	—7.6	—0.2	—11.9	—6.0	—1.1	—0.5	2.8	6.2	6.4	9.7	12.2	4.7	—3.7	—11.3	51.8	U'_N
V	—1.7	0.3	—3.8	—2.2	2.0	1.5	2.5	3.3	3.6	5.1	4.6	0.4	—5.0	—10.0	18.2	V
V'	—0.6	1.8	—3.0	—2.6	0.7	0.0	0.4	1.7	2.3	4.0	4.7	1.5	—3.3	—7.6	15.5	V'
V_B	—1.2	—1.6	—0.9	0.4	1.3	1.5	2.1	1.6	1.4	1.1	—0.1	—1.2	—1.7	—2.4	2.7	V_B

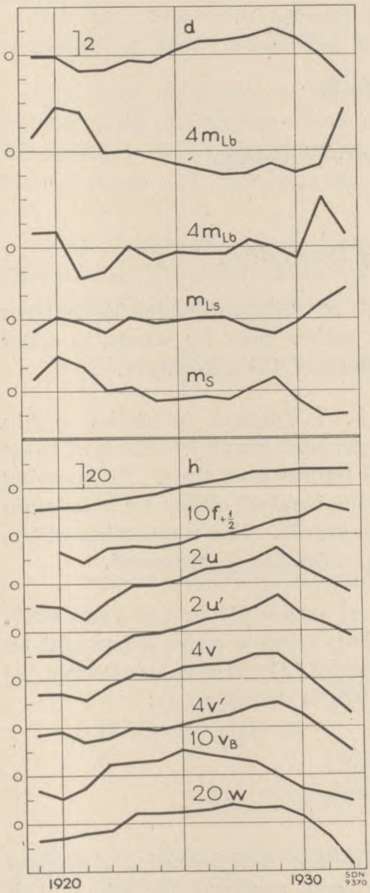
d	—0.2	—0.2	—1.4	—1.3	—0.5	—0.7	0.3	1.0	1.2	1.4	2.0	1.3	—0.2	—2.1	5.9	d
l	—15	4	—5	—8	—1	2	2	4	5	5	7	7	2	—8	93	l
$m_{\mathrm{L}b}$	0.27	0.90	0.81	—0.04	0.00	—0.13	—0.26	—0.38	—0.51	—0.49	—0.28	—0.46	—0.30	0.89	4.98	$m_{\mathrm{L}}{}^{b}$
$\dot{m}_{\mathrm{L}b}$	0.32	0.33	—0.61	—0.51	0.04	—0.26	—0.12	—0.17	—0.15	0.15	—0.01	—0.25	1.05	0.22	0.04	$\dot{m}_{\mathrm{L}}{}^{b}$
$m_{\mathrm{L}s}$	—1.0	0.2	—0.3	—1.0	0.2	—0.2	—0.1	0.2	0.1	—0.7	—1.2	—0.2	1.5	2.7	4.7	$m_{\mathrm{L}s}$
m_{S}	0.9	2.8	2.1	0.0	0.3	—0.8	—0.7	—0.5	—0.7	0.2	1.1	—0.8	—2.1	—1.9	4.7	m_{S}
m_{R}	—25	—11	4	6	7	10	9	8	6	4	1	—2	—7	—16	99	m_{R}
n	—25	—32	—41	—28	—27	—23	—6	4	23	54	95	54	—1	—47	96	n
$\dot{n}$	7	—11	2	14	—3	14	16	12	28	40	8	—57	—56	—12	1	$\dot{n}$
p	2	18	2	—3	0	1	4	4	2	1	0	—3	—13	—22	100	p
p_{D}	14	39	22	2	—1	—3	—5	—8	—7	—6	—4	—8	—14	—18	104	p_{D}
p_{N}	1	15	0	—3	1	2	6	6	4	2	1	—2	—12	—22	99	p_{N}
p'	8	25	3	—3	—1	—1	3	4	2	1	1	—2	—13	—22	99	p'
p^f	49	48	—11	—7	0	1	10	2	—2	5	3	—11	—37	—52	97	p^f
q	6	19	2	—5	5	6	2	2	1	1	0	—4	—12	—23	100	q
q_{B}	—2	23	—1	—14	5	6	2	3	2	2	2	0	—10	—22	98	q_{B}
$f_{+\frac{1}{2}}$	.	—1.4	—2.3	—1.1	—1.0	—1.1	—0.8	—0.1	0.0	0.5	1.1	1.3	2.4	1.9	11.6	$f_{+\frac{1}{2}}$
h	—19	—18	—16	—13	—9	—6	—1	3	7	11	13	14	15	15	165	h
u	—9	—10	—15	—7	—1	—1	1	5	6	9	14	6	1	—4	60	u
u'	—10	—10	—15	—7	—2	—1	1	5	6	9	14	6	2	—3	60	u'
v	—2.6	—2.7	—4.0	—1.2	1.1	0.5	2.2	3.0	3.4	5.0	4.7	1.2	—3.1	—7.4	18.1	v
v'	—1.4	—0.8	—3.0	—2.1	0.0	—0.7	0.2	1.5	2.1	4.0	4.9	2.5	—1.5	—5.1	15.3	v'
v_{B}	—1.2	—1.9	—1.0	0.9	1.1	1.2	2.0	1.5	1.3	1.0	—0.2	—1.3	—1.7	—2.4	2.8	v_{B}
w	—0.7	—0.6	—0.4	—0.3	0.4	0.4	0.4	0.5	0.7	0.6	0.6	0.2	—0.6	—1.9	6.7	w
α	—0.04	0.25	0.33	0.13	0.18	0.08	—0.09	—0.12	—0.18	—0.32	—0.32	—0.08	0.02	0.22	1.71	α
t	—6.5	—5.5	—4.5	—3.5	—2.5	—1.5	—0.5	0.5	1.5	2.5	3.5	4.5	5.5	6.5	0.0	t

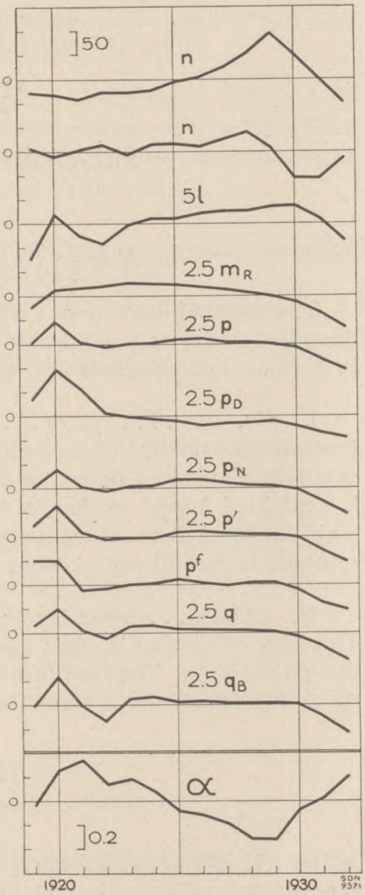
APPENDIX C *(continued)*

GRAPHS OF STATISTICAL MATERIAL USED









The series of figures given in the preceding table are represented graphically above. They are grouped in about the same way as in Appendix A. The unit is the same for all series in each group and the correct relative magnitude of the fluctuations of each series is obtained by multiplication by a round figure, from 2.5 up to 250.

The unit is chosen in such a way that the multiplier in the first series of each group is equal to 1.

		Unit
I.	Money Items	50×10^9 \$
II.	Interest Rates	2%
III.	Prices	50 points
IV.	Physical Quantities	20×10^9 \$ of 1929
V.	Miscellaneous: α	Pareto's unit.

APPENDIX D

SOURCES OF MATERIAL

Note on the System of Description.

The combination and working out of different series have been described as far as possible by formulæ rather than by words, in order to give an unambiguous and short résumé of the procedure.

I. *(1), (2) . . . , (1.1) . . . , (1.11) . . . (figures in italics within brackets and subdivided according to the decimal system)*.[1] Each of these symbols relates to a figure, or a series of figures over time. Its meaning is explained in the section on the variable treated, either by a formula which reduces it to other symbols, or in words. The numbering of the symbols is recommenced when the next variable is discussed.

II. $(1)_{1929}$ . . . indicates the value of series *(1)* in the year 1929. This way of writing is often used, especially when a series is brought on to another basis. The transformation of series *(1)* to the basis of series *(2)* by multiplication by a given ratio for 1929 is described as:

$$(1) \times \frac{(2)_{1929}}{(1)_{1929}}$$

III. **(1)**, **(2)** . . . *(thick upright figures within thick brackets, no decimal subdivision)* refer to the list of sources used, given at the end of this appendix (page 235). For the reader's convenience, authors' names are mentioned in the text. The numbering is continuous for the whole of this appendix.

IV. $\mathbf{(3_{1928})}$ indicates the 1928 volume of the periodical publication **(3)**. (Indication of year and pages is not given for periodicals, except when more or less special tables are used, or when the series is difficult to find.)

V. Ordinary figures are used with the same meaning as elsewhere in this publication (*cf.* Appendix C).

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[1] In a few cases, these figures are in *thick italics* for the sake of clearness.

VI. In a few cases, upright figures in brackets are used to indicate equations; in these cases, the word "equation" is added.

VII. *(Cf.* Appendix A, *General remarks.)* Plain letters are used to indicate both deviations from average and absolute values, provided this does not lead to confusion. A single dash above a letter or a numerical symbol ($\overline{\mathrm{U}}$ or $(\bar{1})$) indicates an average for 1919-1932.

In the description of the symbols, the wording is generally identical with that of the source from which the series is taken.
References to periods are *inclusive* of the last year mentioned.

The series are described in the order of Appendix A.

A. ASSETS HELD BY INDIVIDUALS, VALUE

$$\overline{\overline{\mathrm{A}}} = \frac{\overline{\overline{\mathrm{B}^i}}}{\overline{\overline{m}}_{\mathrm{L}b}} + 0.0156\ \overline{\overline{n\mathrm{C}^i}}$$

0.0156: *cf.* page 33.

A*u*. MONETARY GOLD STOCK

Monetary gold stock of the Federal Reserve Banks; annual average of daily figures. **(8)**

B. LONG-TERM DEBT

B = ***(1)*** + ***(2)*** + ***(3)***.

(1) *Total Private Long-term Debt.* KUVIN **(10)**, page 36.

(2) *Preferred Stocks of All Corporations, Nominal Value, Average during Year.*

(2) = average of two consecutive figures of $\begin{cases} \text{1919-1926: } (2.1) \\ \text{1926-1932: } (2.6) \end{cases}$

$$(2.1) = (2.2) \times \frac{(2.6)_{1926}}{(2.2)_{1926}}$$

$$(2.2)\begin{cases} \text{1919-1925: } (2.21) \times \dfrac{(2.22)}{(2.23)} \\ \text{1926 \quad : } (2.5) \end{cases}$$

$$(2.21)\begin{cases} \text{1919-1922: } (2.3) \\ \text{1923-1925: } (2.4) \end{cases}$$

$$(2.4) = (2.3) + \frac{(2.41)}{(2.42)}$$

$$(2.5) = (2.2)_{1925} + \frac{(2.2)_{1925} - (2.2)_{1919}}{6}$$

(2.22) Gross dividends preferred stock, all industries. KING **(9)**, page 182.

(2.23) Ditto, industries covered by *(2.3)*.

(2.3) Par value preferred stock, 7 industries 1919-1922, 6 industries afterwards. *Ibid.*, page 200.

(2.41) Gross dividends preferred stock, electric light and power industry. *Ibid.*, page 182.

(2.42) Ratio gross dividend over capital, preferred stock, same industry, in 1922. *Ibid.*, p. 203.

(2.6) Preferred stock of all corporations, corrected for corporations not submitting balance-sheets (*cf.* C). STATISTICS OF INCOME **(24)**.

(3) *Public Long-term Debt.*

$$(3) = (3.1) - B^{g}_{s} + (3.2)$$

(3.1) Public debt of the United States, June 30th **(23)**.

(3.2) Total outstanding issues of tax-exempt securities of States, countries, cities, etc. **(23_{1936})**, page 204.

B^b. LONG-TERM DEBT, HELD BY BANKS (NOMINAL VALUE)

$$B^b = 100\,\frac{(1) + (2)}{(4.1)} + 100\,\frac{(3)}{(4.2)}$$

$$(2) = (2.1)\,\frac{(2.2)}{(2.3)}$$

$$(3) = (2.1) - (2)$$

(1) U.S. Government securities held by F.R. Banks; average of daily figures **(8)**.

(2.1) Total investments, all banks, average of figures of three or four call dates.[1] **(8)**

(2.2) U.S. Government obligations, member banks on Dec. 31st *(ibid.)*.

(2.3) Total investments, member banks *(ibid.)*.

(4.1) Average price of U.S. Government bonds, U.S. TREASURY DEPT. **(23)**

(4.2) Corporate bond price index (40 bonds). **(22)** page B 101.

B^e. LONG-TERM DEBT OF PRIVATE ENTERPRISE AND STATE AND LOCAL GOVERNMENTS

$$\Delta B^e = (1) - (1)_{-1} + (2)_{+\frac{1}{2}} - (2)_{-\frac{1}{2}} + (3.2) - (3.2)_{-1}$$

(1) etc.: See B.

$B^g + B^g_s$. TOTAL DEBT OF FEDERAL GOVERNMENT

$\Delta B^g + \Delta B^g_s$: Increase in total interest-bearing debt of the United States Government, Dec. 31st to Dec. 31st **(27)**.

B^i. BONDS HELD BY INDIVIDUALS

$B^i = B - B^b$

$Bi - R^e$. INDEBTEDNESS OF THE FEDERAL RESERVE MEMBER BANKS WITH THE FEDERAL RESERVE BANKS

Monthly and annual figures.

$$Bi - R = \begin{cases} \text{1918-1921: } (1) - (2) \\ \text{1922-1930: } (1) \\ \text{1931-1937: } (1) - (3) \end{cases}$$

$$(2) = (2.1) - (2.1)_{1922}$$

$$(3) = (3.1) - (3.1)_{1929}$$

---

[1] Interpolation of certain figures for 1919 and 1920: *cf.* M''.

(1) B*i*. Bills discounted by the F.R.B.

(2.1) Part of *(1)* secured by Government war obligations; end-of-month figures, and average thereof. **(8)**$_{1924}$, page 43, and previous issues.

(3.1) Excess reserves, all member banks. $(3.1)_{1929} = 0.043$: taken as normal excess reserves for previous years.

(1) and *(3.1)*: average of daily figures. **(8)**

B_s. SHORT CLAIMS

$B_s = (1) + (2) + B_s^g$

(1) Total loans, all banks; average of figures for three or four call dates.[1] **(8)**

(2) Bills discounted + bills bought by the F.R.B.; average of daily figures. **(8)**

B_s^g. SHORT-TERM DEBT OF FEDERAL GOVERNMENT

Treasury bills + Certificates of indebtedness, June 30th (includes Treasury (= war) savings securities up to 1929). **(23)**

C. STOCK OF COMMON CAPITAL OF ALL CORPORATIONS, PAR VALUE (Middle of Year)

S. SURPLUS OF ALL CORPORATIONS (End of Year)

I. *Determination of Stock of Common + Preferred Capital (C″) and of S at End of Year.*

<u>*Ia.* 1926-1932.</u>

$$(C'') = (C''1) + (C''2)$$

$$\left.\begin{aligned}(C''1) &= (C''1.1) \times \{1 + (5.1)\} \\ (C''2) &= (C''2.1) \times \{1 + (5.2)\}\end{aligned}\right\} \text{S treated as } C''$$

$$(5.1) = \frac{\Sigma i\ (5.14)_i\ (5.13)_i}{\Sigma i\ (5.13)_i}$$

---

[1] Interpolation of certain figures for 1919 and 1920: *cf.* M″.

$$(5.14)_i = \frac{(5.11)_i}{(5.12)_i}$$

$$(5.2) \begin{cases} 1928-1932: & (5.25) \\ 1926,\ 1927: & (5.27) \end{cases}$$

$$(5.25) = \frac{\sum i\ (5.24)_i\ (5.23)_i}{\sum i\ (5.23)_i}$$

$$(5.27) = (5.26)\ \frac{(5.25)_{1928}}{(5.26)_{1928}}$$

$$(5.26) = \frac{\sum i\ (5.21)_i}{\sum i\ (5.22)_i}$$

SOURCE: **(24)**

$(C''1.1)$ = C'' of all corporations submitting balance-sheets showing net income.

$(C''2.1)$ = C'' of all corporations submitting balance-sheets showing no net income.

$(5.11)_i$ = Number of corporations with net income, not submitting balance-sheets in income class *i*.

$(5.12)_i$ = Total number of corporations with net income in income class *i*.

$(5.13)_i$ = Total income in income class *i*.

$(5.21)_i$, $(5.22)_i$, $(5.23)_i$, $(5.24)_i$: as $(5.11)_i$, etc., but with *deficit* instead of net income.

I*b*. 1919-1925.

$$C'' + S = (3) \times \frac{(C'' + S)_{1926}}{(3)_{1926}}$$

$$(4.1) = \frac{S}{C'' + S}$$

$$(4.2) = (4.1)_0 - (4.1)_{-1}$$

(4.2) is correlated with Z^c, 1927-1932, and then extrapolated, 1919-1926, on the basis of this relation.

(4.1) extrapolated by using (4.2).

$$S = (4.1) \times (C'' + S)$$

$$C'' = \{1 - (4.1)\} \times (C'' + S)^*$$

---

* The resultant percentage trend in C″ is about equal to that in preferred stocks, which fits in with the fact that there is no apparent trend difference between dividends paid on both kinds of stocks (*cf.* KING **(9)**, pages 189-191).

(3) = Total capitalisation (capital stock + surplus) of 3,144 large corporations, which made 43% of all corporations' profits in 1926. EPSTEIN **(6)**, page 613.

II.

6.1 = C″ — preferred stocks at end of year (*cf.* B, page 211).

$C = \frac{1}{2}\left\{ (6.1) + (6.1)_{-1} \right\}$ *

$\Delta C = C_{+\frac{1}{2}} - C_{-\frac{1}{2}}$

C^i. SHARES HELD BY INDIVIDUALS

$C^i = 0.755\ C$

$$0.755 = \frac{\sum_{1922}^{1932} D}{\sum_{1922}^{1932} D'}$$

D′: *cf.* page 24.

H. CALCULATED HOARDING (H^1)

Residuals (since 1930) of the correlation calculation:

$$M' = 0.043\ (L_w + L_s + E'_F) - 0.076\ t$$

M. TOTAL MONEY

$M = M' + M''$

M′. OUTSIDE CURRENCY

1919-1934: *(1)*

1934-1937: *(2)* — *(3)*

(1) Outside currency, average of 12 end-of-month figures. ANGELL **(2)**, pages 178-179.

(2) Money in circulation, June call date. **(8)**

(3) Vault cash, all banks. **(3)**$_{1935}$, page 101, etc.

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* It has been assumed that $(6.1)_{1918} = (6.1)_{1919}$.

M″. TOTAL DEPOSITS

Average of figures for three or four call dates.

June call date: *(1)*.

Other call dates: *(2)* × *(3)*

$$(2)\begin{cases}1924\text{-}1932: (2.1)\\ 1919\text{-}1923: (2.2) \times (2.3)\end{cases}$$

(1) Total deposits. ANGELL **(2)**, page 175.

(2.1) Total other than inter-bank deposits, all banks in the United States. **(8)**

(2.2) *Ditto*, all member banks. *Ibid.*

(2.3) Average of $\frac{(2.1)}{(2.2)}$ at preceding and following June call date.

(3) Average of $\frac{(1)}{(2)}$ at preceding and following June call date, weighted according to relative distance of time.

P. FEDERAL RESERVE BANKS' HOLDING OF GOVERNMENT SECURITIES, ETC.

$$P = (1) + (2) + (3) - (4) - (5) - (6) - (7) + (8) + (9)$$

(1) Bills bought.
(2) U.S. Government securities.
(3) Other Reserve Bank credit.
(4) Treasury cash holdings.
(5) Treasury deposits with F.R. banks.
(6) Non-member deposits.
(7) Other Federal Reserve accounts.
(8) Treasury currency outstanding.
(9) Correction on $Bi - R^e$ (rediscounts secured by Govt. war obligations, see $Bi - R^e$).

All series: **(8)**, yearly average of daily figures.

R^r. MEMBER BANKS' REQUIRED RESERVES

Yearly average daily figures. Since 1929: + 0.043 (*cf.* $B\,i - R^e$). **(8)**

S. SURPLUS OF CORPORATIONS

See under C.

Z. NET INCOME OF ALL ENTERPRISES AND GOVERNMENT

$Z = Z^c - (1) + E_E - E''_F + (2) + (3)$

(1) Gains and losses on sale of capital assets, corporations (included in Z^c). 1929-1932: KUZNETS **(12)**, page 8, *n.* 7; extrapolated by means of a correlation calculus with Z^c covering 1929-1935.

(2) Non-corporate business savings: Net business savings or losses of agriculture and trade. *Ibid.*, pages 22-3.

(3) Net savings of Government. *Ibid.*, page 8.

Z^c. CORPORATION PROFITS

Statutory net income less statutory deficit.
STATISTICS OF INCOME **(24)**.

N. DEPRECIATION ALLOWANCES

$N = (1) + (2)$

$(2) = (2.11) + (2.12) + (2.2) + (2.3)$

(1) Depreciation and depletion; Repairs, renewals and maintenance, Development expenses and fire losses. FABRICANT **(7)**, page 3.

(2.11) Residences, rented, non-farm: depreciation + major repairs and alterations.

(2.12) Residences, rented, farm: depreciation.

(2.2) ,, , owner-occupied, farm: depreciation.

(2.3) ,, , unallocated: fire losses.

Source of *(2)*: *ibid.*, page 15.

U. VALUE OF PRODUCTION OF CONSUMPTION GOODS AND SERVICES

May be derived from U′, w and p with the help of the equations (1.10), (1.11), (1.12).

U′. CONSUMPTION EXPENDITURE

A. 1919-1929.

U′ = ***(3)*** *All Food, Wearing Apparel and Personal Care* (consisting of ***(1)*** *Non-Manufactured Food* + ***(2)*** *Manufactured Food, Wearing Apparel and Personal Care*) + ***(4)*** *Shelter and Home Maintenance* + ***(5)*** *Other Goods and Services*; ***(6)*** *Corrections.*

(1) *Non-manufactured Food.*

$$(1.3) = (1.1) + (1.2)$$
$$(1) = (1.3) \times (1.4)$$

(1.1) Gross farm income, live-stock and live-stock products **(1)**.

(1.2) Gross farm income, crops, fruits and nuts, and vegetables, 1919 ($\mathbf{1}_{1922}$) and 1924-1932 ($\mathbf{1}_{1931,\ 1938}$); interpolated for 1920-1923 on *(1.21)* gross farm income, fruits and vegetables ($\mathbf{1}_{1928}$) multiplied by $\dfrac{(1.2)_{1924}}{(1.21)_{1924}}$

(1.4) $\dfrac{11.74}{8.35}$; $8.35 = (1.3)_{1929}$, 11.74 = the 1929 value of the consumption of meat, dairy products, vegetables, fruits and nuts.* Warburton **(28)**, page 178.

(2) *Manufactured Food, Wearing Apparel, and Personal Care.*

$$(2.3) = (2.1) \times (2.2)$$
$$(2) = (2.3) \times (2.4)$$

(2.1) Index of production of consumption goods (exclusive of automobiles). Leong **(18)**, page 371.

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* This would point to a distribution margin of 40%.

(2.2) Special index of retail prices, consisting of

(2.21) Cost-of-living index, National Industrial Conference Board **(25)**, Clothing, weight 12 (as in the Board's index).

(2.22) Same index, Food, weight 13.2 (in the Board's index: 33, but 60% — in 1929 — of total food expenditure, according to Warburton, has been taken into account under *(1)*).

$$(2.4) = \frac{22.2}{(2.3)_{1929}}; \quad 22.2 = 33.96 - 11.74. \quad 33.96 = (3)_{1929}$$ according to Warburton, before applying the corrections for *(6.1)* and *(6.5)* mentioned below; 11.74: *cf. (1.4)*.

(3) *All Food, Wearing Apparel and Personal Care.*

$$(3.1) = (1) + (2)$$

$$(3) = \begin{cases} \text{Odd years } (3.2) \\ \text{Even years } (3.1) + \tfrac{1}{2} \left\{ (3.2)_{+1} - (3.1)_{+1} + (3.2)_{-1} - (3.1)_{-1} \right\} \end{cases}$$

(3.2) Warburton's estimate for *(3)*, equally uncorrected for *(6.1)* and *(6.5)*.

(4) *Shelter and Home Maintenance.*

$$(4) = (4.1) + (4.2) + (4.3)$$

$$(4.1) = (4.11) + (4.12)$$

(4.11) Rentals paid for leased non-farm homes. Lough **(20)**, page 243.

(4.12) Rental values of homes on leased farms (Odd years: *ibid.*; other years: straight line interpolation).

(4.2) Home equipment and decoration. Odd years: Warburton; even years interpolated * on Output of consumers' durable finished goods, destined for domestic consumption:

(4.21) Household furniture. Kuznets **(16)**, page 38 *sqq* +

(4.22) House furnishings. *Ibid.* +

(4.23) Household machinery. *Ibid.*

(4.3) Household supplies and operation. Odd years: as *(4.2)*; even years interpolated * on Output of Fuel and Lighting, Gasoline and Lubricating Oils (to household consumers only). *Ibid.*, page 18.

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* *Cf.* formula used for *(3)*.

(5) *Other Goods and Services.*

Odd years: WARBURTON; even years interpolated* on Total Consumer Expenditure on Transportation, Personal, Recreation, Health, Education and Social. DOANE **(4)**, page 67.

(6) *Corrections.*

(6.1) Increases in stocks of consumers' goods, viz. $(w-w_{-1})\,\frac{p}{100}$, are subtracted.

(6.2) Changes in trade margins.

$$(6.2) = \begin{cases} \text{Odd years: } (6.21) \\ \text{Even years: } (6.21) \text{ interpolated}^{*} \text{ on } (6.24) \end{cases}$$

$$(6.22) = \frac{(6.221)}{(6.222)}$$

$$(6.23) = (6.22) - (6.22)_{1929}$$

$$(6.24) = (6.23) \times \frac{(6.21)_{1919}}{(6.23)_{1919}}$$

(6.21) WARBURTON's series for *(6.2)*.

(6.221) Index of wholesale prices, food **(23)**.

(6.222) Index of retail prices, food **(23)**.

(6.3) The value of Government services paid out of the receipts from *indirect taxes* is subtracted.†

$$(6.3) = (6.31) + (6.32) - (6.33)$$

(6.31) Revenue from Customs **(23)**.

(6.32) Miscellaneous internal revenue **(23)**.

(6.33) Revenue from legacy and inheritance duties, included in *(6.32)* **(23)**.

(6.4) E''_F is subtracted.

(6.5) An amount of 0.2 has been subtracted in every year on account of industrial use and preparation of meals (*cf.* WARBURTON).

(6.6) An amount of 0.2 has been subtracted in every year on account of income from urban cows, chickens and garden plots. LEVEN *c.s.*, **(19)**, page 162.

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* *Cf.* formula used for *(3)*.

† *Cf.* WARBURTON **(28)**, page 175. (The correction could only be applied for federal taxes.)

(No correction has been made for the value of imported and exported finished consumption goods; these items could not easily be segregated from Customs statistics, nor can their difference have been of great importance.)

B. 1930-1932.

Following a suggestion kindly made to the author by Mr. H. Barger, of the National Bureau of Economic Research, the following extrapolation has been carried out.

(7) *1930 and 1931.*

$$(7.1) = (7.2) - (7.3)$$

$$(7) = (7.1) \times 0.993$$

$$0.993 = \frac{U'_{1929}}{(7.1)_{1929}}$$

(7.2) Total consumption as estimated by Lough **(20)**, page 28 (Commodities + Intangibles).

(7.3) Net rental values (imputed), *ibid.*, page 243; straight line interpolation for 1930.

(8) *1932.*

$$(8) = (8.1) \times 1.011$$

$$1.011 = \frac{U'_{1929}}{(8.1)_{1929}}$$

(8.1) Consumers' outlay. Kuznets **(17)**, page 85.

U'_D. CONSUMPTION EXPENDITURE ON DURABLE GOODS

Flow of consumers' durable commodities to households and enterprises.

Kuznets **(11)**, page 6.

U'_N. CONSUMPTION EXPENDITURE ON NON-DURABLE GOODS, AND SERVICES

$U'_N = U' - U'_D$.

V. VALUE OF PRODUCTION OF INVESTMENT GOODS

(1) Flow of producers' durable commodities to enterprises +

(2) Volume of total construction.

Kuznets **(11)**, page 96.

V'. VALUE OF PRODUCTION OF PRODUCERS' DURABLE GOODS + NON-RESIDENTIAL CONSTRUCTION

$V' = V - V_B$.

V_B. VALUE OF RESIDENTIAL BUILDING

1920-1932: Value of all residential construction. Extrapolated for 1919 on $v_B \times q_B$.

Wickens and Foster **(30)**, page 2.

d. DIVIDENDS AS A PERCENTAGE OF CAPITAL

$$d = \frac{(1)}{(2)} \times 100$$

$$(1) \begin{cases} 1922\text{-}1932: (1.1) \\ 1919\text{-}1921: (1.2) \end{cases}$$

$$(1.2) = D \times \frac{(1.1)_{1922\text{-}1924}}{D_{1922\text{-}1924}}$$

(1.1) All cash dividends paid out **(24)**.

(2) C + Preferred stock (*cf.* B).

l. WAGE RATE

$$l = \begin{cases} 1921,\ 1923\text{-}1932: (1) \\ 1919,\ 1920,\ 1922: (2) \end{cases}$$

$$(1) = (1.1) \times \frac{100}{(1.1)_{1929}}$$

$$(2) = (2.1) \times \frac{(1)_{1924}}{(2.1)_{1924}}$$

(1.1) Index of hourly earnings in 25 manufacturing industries, all wage-earners **(23)**.

(2.1) Hourly earnings in industry as a whole. Douglas **(5)**, page 205.

m_{Lb}. BOND YIELD

Yield in percentage, 60 issues combined. **(23)**.

m_{Ls}. SHARE YIELD

$$m_{Ls} = \begin{cases} 1926\text{-}1932: (1) \\ 1919\text{-}1925: (2) \end{cases}$$

(2) = Extrapolation on correlation calculus between *(1)*, $\frac{d}{n'}$ and t, 1926-1932.

(1) Share yield of 90 shares. Standard Statistics Co. **(22)**.

n' Index of the price of 90 shares **(22)**, used in the calculation of *(1)*.

m_S. SHORT-TERM INTEREST RATE

Annual average rate on prime commercial paper (4-6 months) in New York. Tinbergen **(26)**, page 157.

m_R. HOUSE RENT

Housing item in Bureau of Labor Statistics cost-of-living index, figures of June (1921: May) **(23)**, brought on to basis 1929 = 100.

D. CASH DIVIDENDS PAID OUT TO INDIVIDUALS

Dividends. Kuznets **(12)**, page 8.

E. URBAN NON-LABOUR INCOME

$$E = D + L_c + K_I + K_R + (E_E - E'_F - E''_F) + E_A.$$

E_A. NET BALANCE OF INTERNATIONAL PAYMENTS

$E_A = (1) - (2) - (3)$.

(1) Property income payments.
(2) Dividends.
(3) Interest.

Kuznets **(12)**, page 8.

E_E. ENTREPRENEURIAL WITHDRAWALS

$E_E = (1) + (2)$.

$$(2) = (2.1) \times \frac{5.1}{(2.1)_{1929}}$$

(1) Withdrawals by entrepreneurs. Kuznets **(12)**, page 8.

(2.1) Aggregate income payments to individuals, Service + Miscellaneous industries. *Ibid.*, page 6.

5.1: Estimated total entrepreneurial withdrawals, in $\$ 10^9$, in these two industries. *Ibid.*, page 9.

E'_F. FARMERS' CONSUMPTION EXPENDITURE

$E'_F = (1) - E''$

(1) Withdrawals by farm operators. KUZNETS **(13)**, page 22.

E''_F. FARMERS' CONSUMPTION OF HOME-PRODUCED GOODS

$E''_F \begin{cases} \text{1924-1932: } (1)-(2) \\ \text{1919-1923: extrapolated on the basis of a correlation} \end{cases}$
calculus between E''_F and p^f over 1924-1936.

(1) Gross income from agricultural production **(23)**.

(2) Cash ,, ,, ,, ,, **(23)**.

G. CAPITAL GAINS REALISED

$G \begin{cases} \text{1919-1929: WARBURTON (29), page 86.} \\ \text{1930-1932: Extrapolated on equation (5.3).*} \end{cases}$

K_I. INTEREST PAID OUT TO INDIVIDUALS BY OTHERS THAN INDIVIDUALS

$K_I = (1) - (2)$

$$(2) \begin{cases} (2.1)\text{: 1929-1932} \\ (2.2)\text{: 1919-1928} \end{cases}$$

$$(2.2) = (2.21) \times \frac{(2.1)_{1929}}{(2.21)_{1929}}$$

---

* There is reason to assume that capital losses reported for these years seriously underestimated the losses really suffered as the statistics show losses only in cases where there is other income from which to deduct them.

(1) Interest. Kuznets **(12)**, page 8.

(2.1) Interest on individuals' mortgages. Kuznets **(15)**, page 184.

(2.21) Non-business interest. Leven **(19)**, page 153.

K_R. RENTS PAID OUT TO INDIVIDUALS

$$Kr = (1) - (2)$$

$$(2) \begin{cases} (2.1) : 1919\text{-}1927 \\ (2.2) : 1928 \\ (2.3) : 1929 \\ (2.4) : 1930\text{-}1932 \end{cases}$$

$$(2.2) = (2.21) \times \frac{(2.1)_{1927}}{(2.21)_{1927}}$$

$$(2.3) = (2.31) \times \frac{(2.1)_{1927}}{(2.31)_{1927}}$$

$$(2.4) = (2.41) \times \frac{(2.3)_{1929}}{(2.41)_{1929}}$$

(1) Rents. Kuznets **(12)**, page 8.

(2) Imputed rents.

(2.1) Imputed income from owned non-farm homes. Leven **(19)**, page 153.

(2.21) Gross income real estate and holding companies **(24)**$_{1927}$, page 331, *etc.*

(2.31) Rentals. Warburton **(28)**, page 178.

(2.41) Net rents and royalties. Kuznets **(14)**, page 5.

L_c. CORPORATION MANAGERS' SALARIES

Total compensation of corporate officers **(24)**.

L_s. LOWER SALARIES

$L_s = (1) - L_c$

$$(1) \begin{cases} 1919\text{-}1925: (1.1) \\ 1926\text{-}1929: (1.2) \\ 1930\text{-}1932: (1.3) \end{cases}$$

$$(1.3) = (2) \times \frac{(3)_{1929}}{(2)_{1929}} \times L_c + (4)$$

$$(2) = \frac{(2.1)}{(2.2)}$$

$$(3) = \frac{(1) - (4)}{L_c}$$

(1.1) Total salaries drawn by employees from all industries. King **(9)**, page 138.

(1.2) Compensation of employees, salaries. Leven **(19)**, page 155.

(2.1) Total salaries in selected industries: mining, manufacturing, construction and transportation. Kuznets **(15)**, page 47.

(2.2) Total compensation of corporate officers in same industries. *Ibid.*, page 50.

(4) Compensation of employees, Government service. *Ibid.*, page 192.

L_w. WAGES

$L_w = (1) - (2) - L_s - L_c$

(1) Employees' compensation. Kuznets **(12)**, page 8.

(2) = Series *(2)* in the description of E_E.

n. SHARE PRICE

Annual average prices of 419 common stocks.
1926 = 100. Standard Statistics Co. **(23)**.

$$\dot{n} = n_{+\frac{1}{2}} - n_{-\frac{1}{2}}$$

$n_{+\frac{1}{2}}, n_{-\frac{1}{2}}$: Average of monthly figures of n from July to June of next year.

p. COST OF LIVING

National Industrial Conference Board, annual average. **(25)**. Brought on to basis 1929 = 100.

p_D. PRICE OF DURABLE CONSUMPTION GOODS

Relation (×100) between *(1)* flow of consumers' durable commodities to households and enterprises, *current prices*, and *(2) idem, 1929 prices*. Kuznets **(11)**, page 6.

p_N. PRICE OF NON-DURABLE CONSUMPTION GOODS, AND SERVICES

$$p_N = \frac{p - 0.1265\, p_D}{0.8735}$$

$$0.1265 = \frac{\overline{U'_D}}{\overline{U'}} \; ; \quad 0.8735 = \frac{\overline{U'_N}}{\overline{U'}}$$

It makes only a negligible difference if a smaller weight, more in accordance with the composition of the cost-of-living index used, is taken for U'_D.

p'. COST OF LIVING, EXCLUDING RENT

$$p' = \frac{p - 0.20\, m_R}{0.80}$$

0.20 = weight of m_R in p, index used.

p^f. INDEX OF FARM PRICES

Department of Agriculture series. Calendar year. Brought on to basis 1929 = 100. **(1)**.

Further Data relating to Section (3.4):

Commodity *(j)*	U_j Millions of dollars	φ_j	σ_j	η_j
Wheat	815	0.40	100	0.08*
Maize ("Corn")	2212	1.00		0.48*
Oats	510	0.33		0.57*
Barley	138	0.33[1]		0.47*
Rye	20			2.31*
Buckwheat	11			1.10*
Cotton	487	0	202	0.11*
Fruits & nuts	576	1.00	168	0.33††
Vegetables	614			
Potatoes	401			0.31*
Poultry & eggs	1101	1.00	22	0.80††
Dairy products	1894	1.00	2	
Cattle, sheep and lambs	1156	1.00	36	0.49†
Hogs	1329			0.81†
Hay	1040	1.00	132	0.55*

Sources:

U_j: **(1)**

φ_j: *cf.* section (3.4)

σ_j: **(1)**

η_j:
- * Schultz **(21)**, page 548, etc. Approximate median of 6 observations, post-war data.
- † *Ibid.*, page 583.
- †† Estimates.

q. PRICES OF CAPITAL GOODS

$$q = \frac{(1)}{(2)} \times 100$$

(1) Total Flow of Finished Durable Commodities at current prices Kuznets **(11)**, page 6.

(2) *Ditto* at 1929 prices.

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[1] Taken as for oats.

q_B. CONSTRUCTION COSTS

Engineering News-Record, annual average **(23)** brought on to basis 1929 = 100.

$f_{+\frac{1}{2}}$. AGRICULTURAL SUPPLY AVAILABLE FOR THE UNITED STATES MARKET (Crop years)

$$f_{+\frac{1}{2}} = \{\boldsymbol{(1)} + \boldsymbol{(2)} - \boldsymbol{(3)}\} \times \frac{12.69}{\{(1) + (2) - (3)\}_{1929}}$$

$$\left.\begin{array}{l} (1)\ \text{Production} \\ (2)\ \text{Stocks} \\ (3)\ \text{Exports} \end{array}\right\} \text{ of agricultural products}$$

12.69 = value ($\$10^9$), in 1929, of $(1) + (2) - (3)$.

(1) *Index of Farm Production*, 1924-1929 = 100.

(2)
$$(2) = (2.1) \times \frac{1.116}{(2.6)} \times \frac{(1)_{1923/5}}{(2.1)_{1923/5}}$$

$$(2.1) = \frac{(2.11) \times (2.21) + (2.12) \times (2.22)}{(2.21) + (2.22)}$$

$$1.116 = (2.3) \times \frac{(2.21) + (2.22)}{(2.22) \times \dfrac{(2.4)}{(2.4) + (2.5)}}$$

$$(2.3) = (2.31) \times (2.32)$$

$$(2.5) = (2.51) \times (2.52)$$

(2.11) Index stocks of raw materials, foodstuffs (Yearly average, 1923-1925 = 100), July (Dept. of Commerce) **(25** $_{1928,\ 1932,\ 1936}$**)**.

(2.12) *Ditto*, textile materials.

(2.21) } Weights of *(2.11)* and *(2.12)* in the index of commodity stocks
(2.22) } **(25** $_{1928}$**)**, August, page 20.

(2.31) American cotton carry-over.

(2.32) Season average prices of cotton, received by farmers.

(2.4) Farm value of gross production of cotton.

(2.51) Silk imports, quantity.

(2.52) Silk price. **(25)**

(2.6) Gross income from farm production.

(2.31), *(2.32)*, *(2.4)*, *(2.51)*, *(2.52)*, *(2.6)* represent the average of the series mentioned over the years 1923-1925.

(3) $$(3) = (3.1) \times \frac{100}{(3.1)_{1924/9}} \times \frac{(3.2)}{(3.3)}$$

(3.1) Index agricultural exports, 1910/11 to 1913/14 = 100.

(3.2) Value agricultural exports.

(3.3) Gross income from farm production.

(3.2) and *(3.3)* represent the average of the series mentioned over the years 1924-1929.

Source, except where mentioned otherwise: **(1)**

h. STOCK OF DWELLING-HOUSES (in $\$10^9$ of 1929)

$$h = \frac{(1)}{0.142}$$

$$0.142 = \frac{(2)}{0.92\left(\sum_{1920}^{1929} v_B + \frac{1}{4}(v_B)_{1930}\right)}$$

0.92: *cf.* text, page 26.

(1) Number of houses, in millions. Tinbergen **(26)**, page 156.

(2) Increase in number of houses (in millions), between the censuses of 1920 (January 1st) and 1930 (April 1st). **(23)**

u. QUANTITY OF CONSUMPTION GOODS AND SERVICES PRODUCED

$$u = \frac{U}{p}.$$

---

u'. QUANTITY OF CONSUMPTION GOODS AND SERVICES SOLD TO FINAL CONSUMERS

$$u' = \frac{U'}{p}.$$

---

v. QUANTITY OF INVESTMENT GOODS PRODUCED

Source as for V, but " at 1929 prices ".

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v'. QUANTITY OF PRODUCERS' DURABLE GOODS + NON-RESIDENTIAL CONSTRUCTION PRODUCED

$$v' = v - v_B.$$

---

v_B. VOLUME OF RESIDENTIAL BUILDING

1920-1932 : $v_B = \frac{V_B}{q_B}$

1919 : Extrapolated on construction contracts awarded, floor space of buildings, residential. Dodge **(23)**.

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w. STOCK OF CONSUMERS' GOODS, QUANTITY AT END OF YEAR

$$w = \frac{W}{p}.$$

$$W = (1) \times \frac{(2)_{1929}}{(1)_{1929}}$$

$$(1) = \{(1.1) - (\overline{1.1})\} \times \frac{\{(2)_{1929} - (2)_{1933}\} : (2)_{1929}}{\{(1.1)_{1929} - (1.1)_{1933}\} : (1)_{1929}} + (\overline{1.1})$$

(1.1) Index department store stocks **(25)**.

(2) Value of all retail stocks. Census **(23)**.

All figures, also for p, at end of year or last month of year.

α. DISTRIBUTION OF INCOME

According to Pareto's law:

$$N_x = Ax^{-\alpha}$$

where N = number of persons with income > x,

x = income (as shown by tax returns);

A and α are constants for any given moment.

Since this formula is supposed to hold good for any value of x (within a certain range):

$$N_{x_1} = Ax_1^{-\alpha}$$

$$N_{x_2} = Ax_2^{-\alpha}$$

and, by division:

$$N_{x_1}/N_{x_2} = (x_1/x_2)^{-\alpha}$$

$$-\alpha = \frac{\log N_{x_1} - \log N_{x_2}}{\log x_1 - \log x_2}.$$

$$x_1 = \$25{,}000; \quad x_2 = \$100{,}000.$$

Source: **(23)**.

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NAME INDEX

SUBJECT INDEX

APPENDIX A

LIST OF VARIABLES INCLUDED IN SYSTEM

I. MONEY ITEMS

(i) *Capital Items.*

More information: Section	Page		
(4.7)	101	A	= Assets held by individuals, value.
(4.4)	82	Au	= Gold stock.
(4.1)	75	B	= Total long-term debt.
(4.5)	88	B^{b}	= Bonds held by banks, nominal value.
(4.1)	75	B^{e}	= Bonds issued by private enterprise, and local and State governments.
(4.1)	77	B^{g}	= Bonds issued by United States Federal Government.
(4.7)	101	B^{i}	= Bonds held by individuals, nominal value.
(4.4)	82	$(B^{i}\text{-}R^{e})$	= Indebtedness of banks to Federal Reserve Banks.
(4.3)	81	B_{s}	= Short claims.
(4.1)	77	B_{s}^{g}	= Short claims issued by United States Federal Government.
(4.2)	79	C	= Capital stock of corporations, nominal value.
(4.7)	101	C^{i}	= Shares held by individuals, nominal value.
(4.6)	93	H	= Hoarding.
(4.4)	82	M	= Total money.
(4.6)	92	M'	= Outside currency.
(4.6)	100	M''	= Deposits.
(4.4)	82	P	= Federal Reserve Banks' holding of securities, etc.
(4.4)	82	R^{r}	= Member Banks required reserves.
(1.9)	29	S	= Surplus of corporations.
(4.6)	98	VC	= Vault cash.

(ii) *Incomes.*

Section	Page		
(5.1)	115	D	= Dividends.
(1.3)	25	E	= Income of urban non-workers.
(1.3)	25	E_{A}	= Net income from abroad.
(5.2)	116	E_{E}	= Entrepreneurial withdrawals.
(2.2)	44	E'_{F}	= Farmers' consumption expenditure.
(2.3)	44	E''_{F}	= Farmers' consumption of home-produced goods.
(5.3)	116	G	= Capital gains realised.
(5.4)	117	K_{I}	= Interest payments.
(5.5)	119	K_{R}	= Rent payments.
(5.6)	119	L_{c}	= Corporation managers' salaries.
(5.7)	120	L_{s}	= Lower salaries.
(5.8)	121	L_{w}	= Wages.
(5.10)	123	Z	= Net income of all enterprises and Government.
(5.11)	125	Z^{c}	= Net income of corporations.

(iii) *Other flows.*

Section	Page		
(5.9)	121	N	= Depreciation allowances.
(1.10)	30	U	= Value of production of consumption goods and services.
(5.10)	123	U^{e}	= Exports, value.
(5.10)	124	U^{i}	= Imports, value.
(2.1)	35	U'	= Total consumption expenditure.
(2.1)	40	U'_{D}	= Consumption expenditure on durable goods.
(2.1)	40	U'_{N}	= Consumption expenditure on non-durable goods and services.
(1.13)	31	V	= Value of production of investment goods.
(1.15)	32	V'	= Value of production of producers' durable goods + non-residential construction.
(1.16)	32	V_{B}	= Value of residential building.

II. PRICES

Section	Page		
(1.2)	24	d	= Dividends as a % of capital.
(3.1)	54	l	= Wage rate.
(4.1)	76	m_{Lb}	= Bond yield.
(1.7)	27	m_{Ls}	= Share yield.
(4.1)	76	m_{S}	= Short-term interest rate.
(3.2)	56	m_{R}	= House rent.
(4.8)	106	n	= Share price.
(1.8)	29	p	= Cost of living.
(2.1)	42	p_{D}	= Price of durable consumption goods.
(2.1)	42	p_{N}	= Price of non-durable consumption goods and services.
(3.3)	57	p'	= Cost of living, excluding rent.
(3.4)	59	p^{f}	= Farm prices.
(3.5)	63	q	= Price of investment goods.
(3.6)	71	q_{B}	= Construction costs.

III. PHYSICAL QUANTITIES

Section	Page		
(3.4)	59	f	= Agricultural supply available for the United States market.
(1.4)	26	h	= Stock of dwelling-houses.
(1.11)	31	u	= Quantity of consumption goods and services produced.
(5.9)	121	u^{e}	= Exports, quantity.
(5.9)	121	u^{i}	= Imports, quantity.
(1.12)	30	u'	= Quantity of consumption goods and services sold to final consumers.
(1.14)	31	v	= Volume of production of investment goods.
(2.4)	45	v'	= Volume of production of producers' durable goods + non-residential construction.
(2.5)	45	v_{B}	= Volume of residential building.
(2.6)	50	w	= Stocks of finished consumption goods, volume.

IV. MISCELLANEOUS

Section	Page		
(2.1)	36	α	= Distribution of income.
		t	= Time.

General Remarks.

Time unit is 1 year (except where stated otherwise).

A · over a variable indicates the derivative in respect to time: $\dot{n} = \frac{dn}{dt}$.

A Δ before a variable indicates the increase over the preceding unit of time: $\Delta B = B - B_{-1}$.

A $\int$ before a variable indicates its cumulation over time: $\int Z_t = Z_{1919} + Z_{1920} + \ldots . Z^t$.

The absolute magnitude of a variable is indicated by $\bar{\bar{\ }}$: $\bar{\bar{Z}}$.

The average magnitude of a variable is indicated by $\bar{\ }$: $\bar{Z}$.

The deviation from average of a variable is indicated by a plain letter: Z.

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